Modelling and control summaries

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USE OF MATLAB 1 – solving ODEs

OVERVIEW: These notes gives a very narrow view of MATLAB and thus demonstrate how to do a limited number of things. In general students are encouraged to become flexible independent learners using the provided MATLAB helps as this is a widely required skill in industry.

SOLVING ODEs: This note will consider only ODEs with constant coefficients and also only demonstrates 1st and 2nd order.

$$A\frac{dx}{dt}+Bx=f(t); \quad x(0)=x0$$

$$A\frac{d^2x}{dt^2}+B\frac{dx}{dt}+Cx=f(t); \quad x(0)=x0; \quad \dot{x}(0)=\dot{x}0$$

1. How do I use MATLAB to find an analytic solution? [**Covered here**]
2. How do I use MATLAB to find a numerical solution and thus one I can plot? [**Cover elsewhere]**

ANALYTIC SOLUTIONS: These make use of the symbolic toolbox so you need to be sure you have this. (Type help in the command window and a list of available toolboxes will be listed)

The BASIC MATLAB command is **dsolve**. Students need to learn how to enter their ODE in the required format. This is best illustrated by a number of examples.

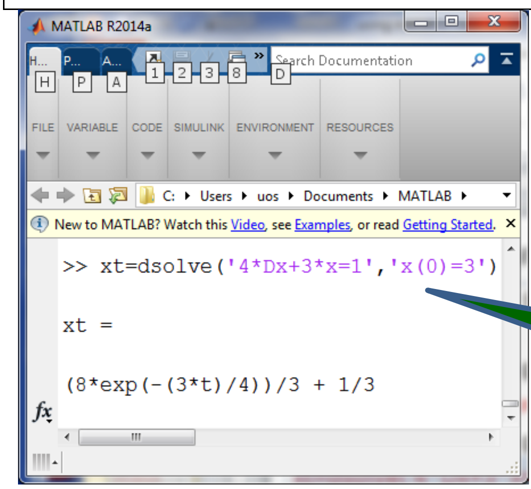
$4\frac{dx}{dt}+3x=t^2+\cos t; \quad x(0)=3$	xt=dsolve('4*Dx+3*x=t^2+cos(t)','x(0)=3')
$4\frac{dx}{dt}+3x=1; \quad x(0)=3$	xt=dsolve('4*Dx+3*x=1','x(0)=3')
$4\frac{dx}{dt}+3x=1; \quad x(1)=2$	xt=dsolve('4*Dx+3*x=1','x(1)=2')

Note that any 'multiply' is explicit.

Initial condition

Solution

ODE in required format (inside quotes)



Note how colour changes to indicate a correct formatting.

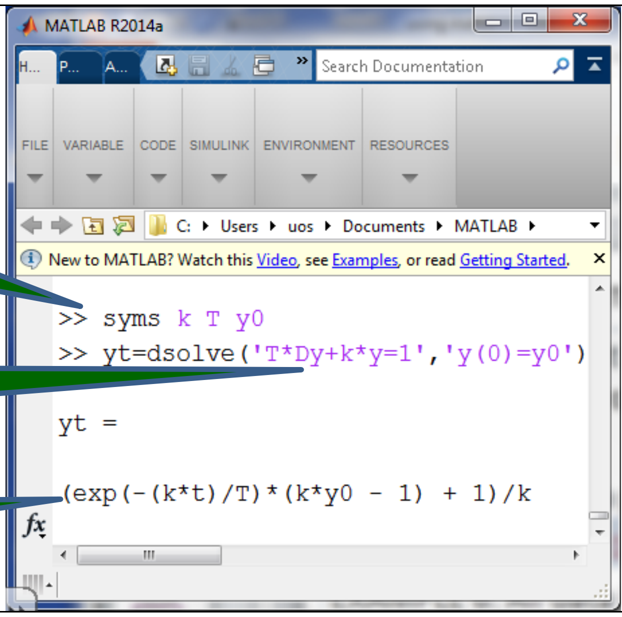
MORE ADVANCED USE: What if not all initial conditions are known or if the ODE coefficients are not known? These can be replaced by symbolic variables and **dsolve** will still work.

$$T\frac{dy}{dt}+ky=1; \quad y(0)=y0$$

Create symbolic variables as required

Use symbolic variables in the dsolve command.

Solution in terms of the unknowns



Second order ODE: These are demonstrated by example and use the equivalent syntax to 1st order, but you need 2 initial conditions to get a full solution.

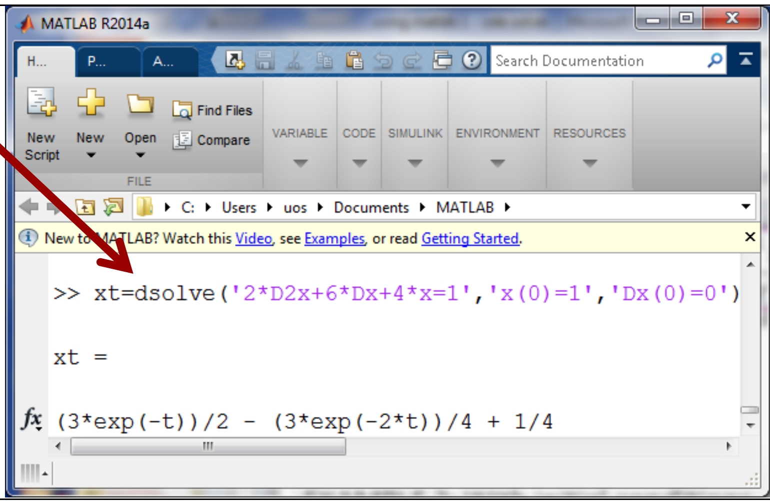
The BASIC MATLAB command is **dsolve**. Students need to learn how to enter their ODE in the required format. This is best illustrated by a number of examples.

$$2\frac{d^2x}{dt^2}+6\frac{dx}{dt}+4x=1$$	xt=dsolve('2*D2x+6*Dx+4*x=1') NO INITIAL CONDITIONS PROVIDED so solution has underdetermined coefficients.
$$2\frac{d^2x}{dt^2}+6\frac{dx}{dt}+4x=1; \quad x(0)=1, \ \dot{x}(0)=0$$	xt=dsolve('2*D2x+6*Dx+4*x=1','x(0)=1','Dx(0)=0')

Students should notice the syntax adopted by MATLAB of

D2x to represent d^2x/dt^2 and Dx to represent dx/dt

Equivalently students can use D2z for d^2z/dt^2 and so forth – see example below.



A 3rd order example is given to demonstrate the basic principle and notation extends.

$$2\frac{d^3z}{dt^3}+2\frac{d^2z}{dt^2}+6\frac{dz}{dt}+4z=\sin(t); \quad z(0)=1, \ \dot{z}(0)=0; \quad \ddot{z}(0)=1$$

zt=dsolve('2*D3z+2*D2z+6*Dz+4*z=sin(t)','z(0)=1','Dz(0)=0','D2z(0)=1')