Modelling and control summaries

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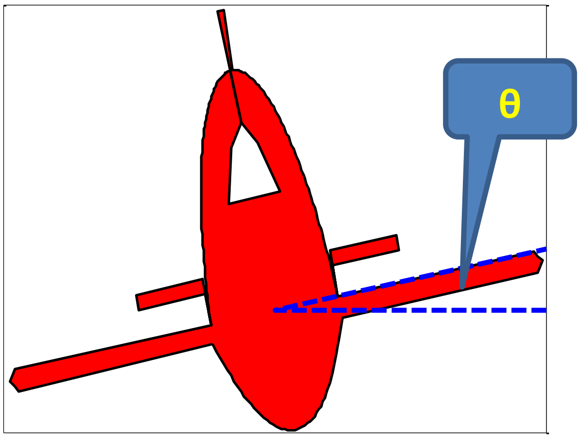
Margins 20 – roll control of an aircraft with a lead

BACKGROUND: A typical requirement for an aircraft is to control the bank angle. The link between the bank angle θ and the movement u of the ailerons can be approximated by models similar to the following:

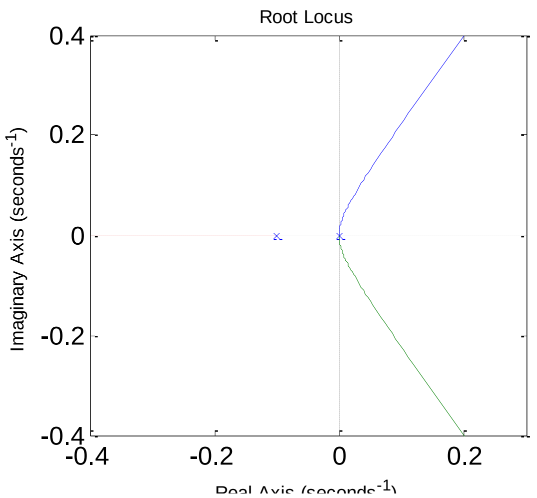
$$\theta = \frac{C}{s^2(s+a)} u = G(s)u(s)$$

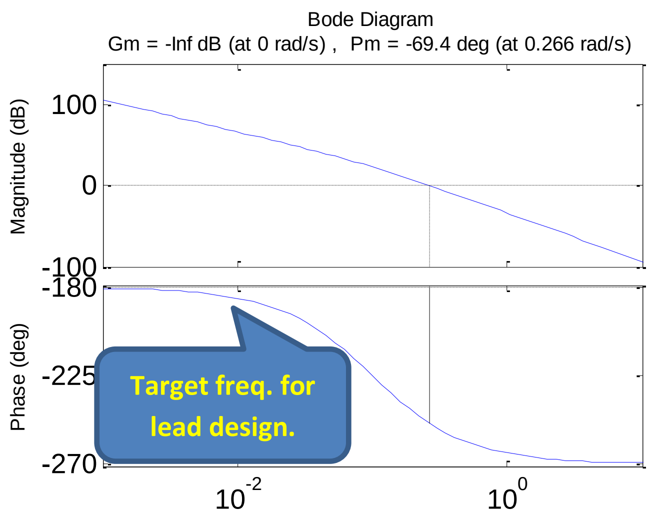


SYSTEM ANALYSIS: The open-loop system includes two integrators and thus will give zero offset to step demands, as long as it is closed-loop stable. However, a quick view of the root-loci and Nyquist diagrams (below with *C=0.01, a=0.1*) indicates that with simple proportional feedback, this system will be closed loop unstable.

- There are always 2 poles in the RHP.
- The phase is always below -180^o and thus one can not obtain a positive phase margin.

SUMMARY: A lead compensator is needed to increase the phase above -180^o, and obviously this will also draw some of the root-loci into the LHP. Here we need a lead pole/zero to be around 0.01! Likely one will need a lot of phase rotation, so aim for ratio of pole to zero close to 10.





LEAD COMPENSATION: A lead compensator takes the following form.

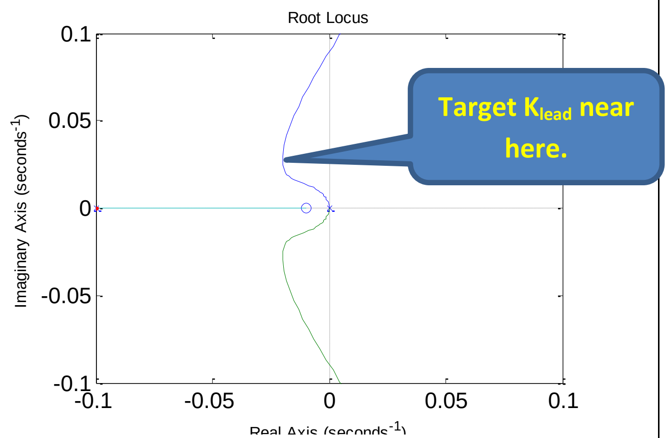
$$M(s) = K_{lead} \frac{s + w/\sqrt{r}}{s + w\sqrt{r}}; \quad r > 1$$

Using w=0.01 and r=10 gives:

$$M(s) = K_{lead} \frac{s + 0.0032}{s + 0.032};$$

The gain can be selected to ensure effective damping from a root-loci plot, or otherwise.

K_{lead}=0.01 gives PM=40°.



Investigate efficacy of the design with a GUI – aeroplanerollgui.m

The GUI allows the user to experiment with:

- Different parameters for the plane, ***C, a***.
- Different choices of lead parameters, **w, r, K_{lead}**.

For simplicity, the demanded roll angle is fixed at 0.5 radians as linearity is assumed.

User changes parameters and uses the push buttons to see the impact.

$$G = \frac{C}{s^2(s+a)}$$

$$M(s) = K_{lead} \frac{s + w/\sqrt{r}}{s + w\sqrt{r}}; \quad r > 1$$