# Modelling and control summaries

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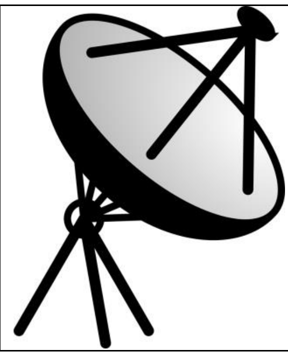
## Offset 13 – radar dish with lag compensation

**BACKGROUND:** A typical requirement for a radar dish is to track non-constant signals which are often close to ramp signals (following a satellite for example). This note presents a case study based on such an example. A simple model for a radar dish is equivalent to a DC servo, thus position θ is governed by a model of the form:

$$\theta = \frac{C}{s(Ts+1)}u = G(s)u(s)$$



**LAG COMPENSATION:** The open-loop system includes an integrator and thus will give zero offset to step demands. However, to track ramps an increase in low frequency gain is required; this is obtained with a lag compensator.

$$M(s) = K_{lag}\frac{s+a}{s+\frac{a}{r}}$$

**REMARK:** The parameter $K_{lag}$ is selected to give a 60 degree phase margin when a=0.

**LAG COMPENSATION:** The parameter $K_{lag}$ is selected to give a 60 degree phase margin when a=0. Hence, find the frequency $w_g$ where the phase is -120° and then find the gain to make this the gain cross-over frequency. The choice of 'a' is then automatic.

$$\angle G = -120^{o} \quad \Rightarrow \quad w_g T = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad w_g = \frac{1}{T\sqrt{3}}$$

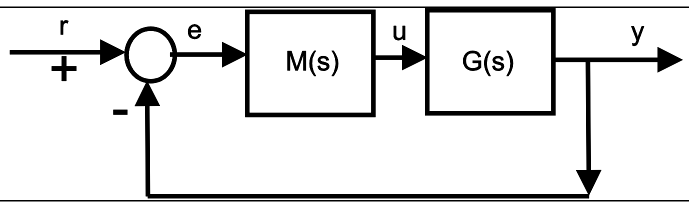
$$|G(jw_g)M(jw_g)| = 1 \quad \Rightarrow \quad K_{lag} = \frac{1}{|G(jw_g)|} = \frac{2}{3CT}$$

**NOTE: Choose** $a = \frac{w_g}{10}$

**DEFINITION OF CLOSED-LOOP SYSTEM OFFSET TO A UNIT RAMP TARGET**

Using the results of earlier notes but using $R(s)=1/s^2$ the error is given as:

$$e_{ramp} = \frac{1}{1+GM}\frac{1}{s^2}$$




**Hence:** $$offset = \lim_{s\to 0}\frac{1}{1+G(s)M(s)}\frac{1}{s} = \lim_{s\to 0}\frac{1}{sG(s)M(s)} = \frac{1}{CK_{lag}r} = \frac{3CT}{2Cr} = \frac{3T}{2r}$$

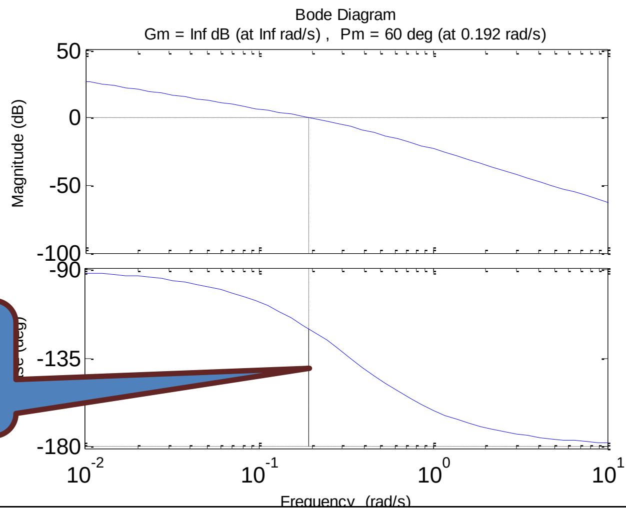
**The offset to a ramp can be reduced by increasing 'r'!**

**The following will demonstrate this automated design for various values of C,T and r!**

**EXAMPLE 1:** Choose arbitrary values of C, T, determine $K_{lag}$ and find the margins.

$$\{C=1, T=3\} \quad \Rightarrow \quad w_g = 0.19; \quad K_{lag} = 0.22;$$

Phase margin is exactly $60^{o}$ as expected.

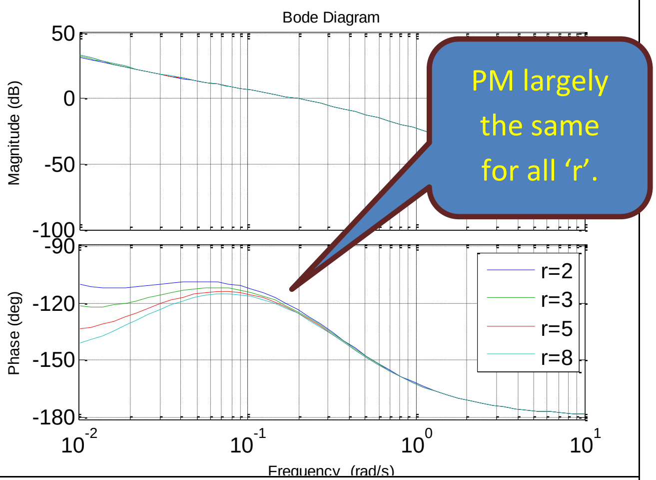



**EXAMPLE 1 continued:** Next consider the impact of adding the lag component by looking at different choices for 'r', that is {2,3,5,8}.

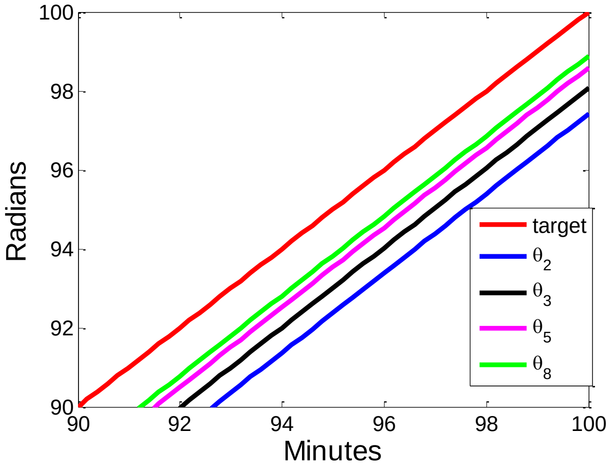
Phase margin is largely unaffected, but the low frequency gains are and hence so are the corresponding offsets to a unit ramp:

$$offset = \frac{3T}{2r} = \{4.5 \quad 3 \quad 1.8 \quad 1.13\}$$

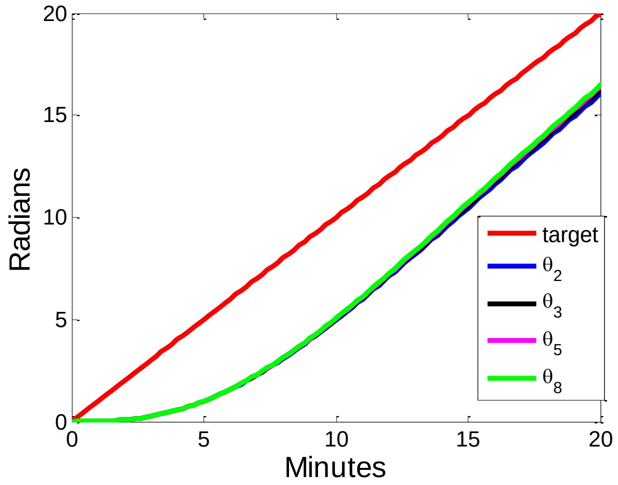
**Note increasing T also increases offset!**



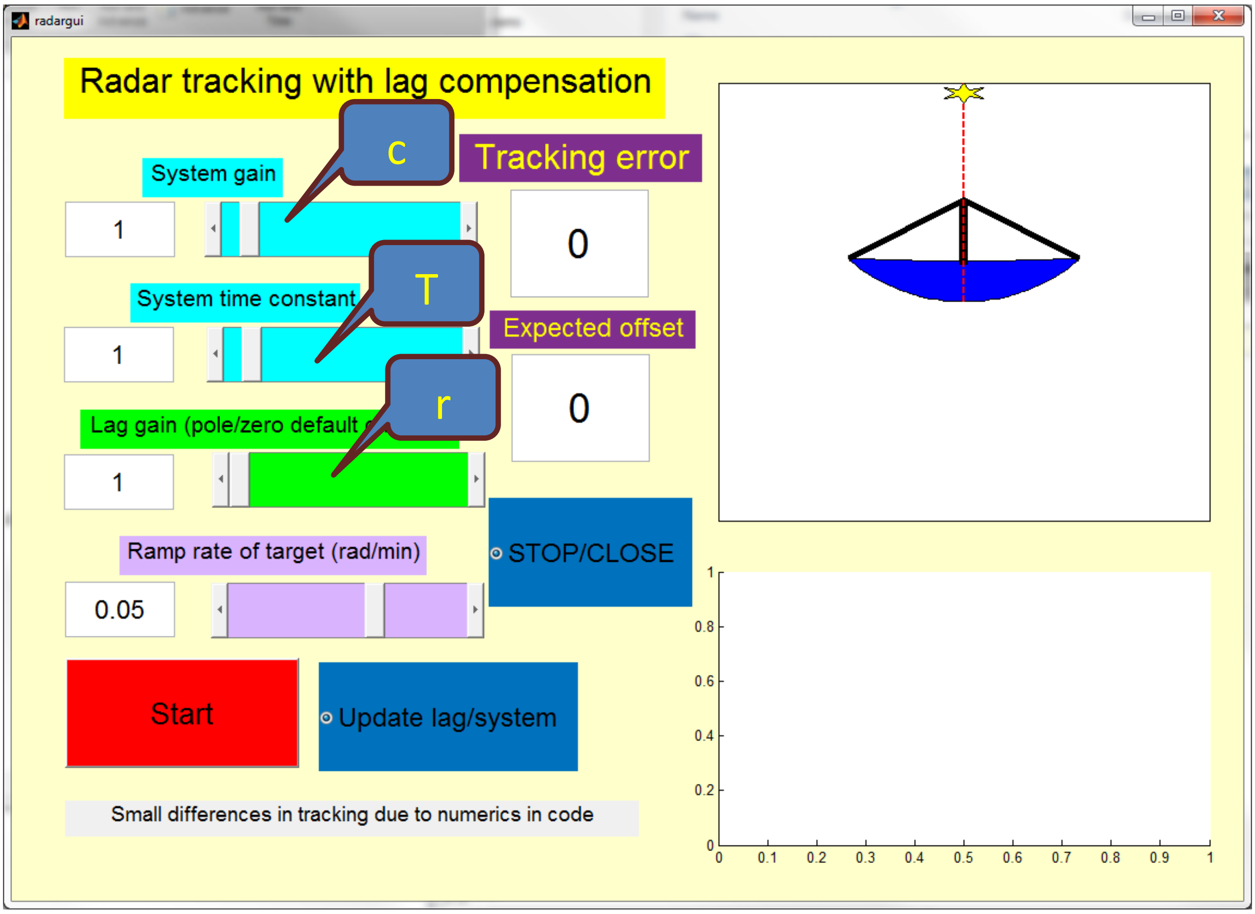

**Asymptotic behaviour is very different – increasing 'r' has clearly reduced the steady-state offset.**




**Transient behaviour is largely the same irrespective of the choice of 'r' as expected from the bode diagrams above.**



# SEE RADARGUI.M TO EXPERIMENT WITH DIFFERENT SCENARIOS




**Note $K_{lag}$ has a default value in this GUI as described in this note. Lag gain means ratio of low frequency to high frequency gain.**