What is the correct gain and phase of the output?

$$\frac{dy}{dy} + 4y = 2\sin 3t \quad \Rightarrow \quad y = A\sin(3t+\varphi) + Be^{-4t}$$

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For each pair below, find the gain and phase of G(jw) in terms of w and hence find the asymptotic value of y(t) when Y(s)=G(s)U(s).

$$G(s) = \frac{8}{s+7} \qquad G(s) = \frac{1+2s}{s+2} \qquad G(s) = \frac{4}{s+0.3} \qquad G(s) = \frac{3}{(s+2)(s+5)}$$

$$u = \sin(6t) \qquad u = \sin(4t) \qquad u(t) = \cos(0.1\sqrt{3}\, t) \qquad u(t) = \sin(2t)$$

Find the gain and phase of the following transfer functions.

$$G(s) = \frac{(s+2)}{(s+3)}; \quad G(s) = \frac{(s+2)}{(s+3)(s+4)}; \quad G(s) = \frac{(s+1)(s+2)}{(s+3)(s+4)}; \quad G(s) = \frac{4}{s(s+4)}$$

$$G = \frac{5}{s-2\sqrt{3}}; \quad G = \frac{1-s}{s+3}; \quad G = \frac{3-s}{s(s^2-1)}$$

Sketch the Bode diagrams, and hence the Nyquist diagrams, for the following systems.

$$G(s) = \frac{(s+2)}{(s+3)}; \quad G(s) = \frac{(s+2)}{(s+3)(s+4)}; \quad G(s) = \frac{(s+20)}{(s+3)^2(s+4)}; \quad G(s) = \frac{4}{s(s+4)}$$

$$G = \frac{5}{s-2\sqrt{3}}; \quad G = \frac{1-s}{s+3}; \quad G = \frac{3-s}{s(s^2-1)}; \quad G(s) = \frac{s+2}{s(s+1)^2(s+5)}; \quad G(s) = \frac{s+20}{s(s+1)^2(s+5)}$$

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1. How do you interpret the information in the Bode plots ?
2. What is frequency response?
3. How do you do a quick sketch of the bode plots?
4. Where do the sketches need to be accurate?
5. Why do gain plots always go to zero as frequency increases?
6. Why does phase lag tend to increase with frequency?
7. Can you compute asymptotes reliably? Do you understand how the asymptotes are derived?
8. Would you be comfortable sketching a Bode plot within just a few minutes.
9. Extract data from the Bode plot to evaluate the output response at a given frequency.

Give an explanation of resonance and illustrate with 3 examples. Include the Bode diagrams.

$$\frac{1}{s^2+2s+2};\quad \frac{1}{s^2+s+2};\quad \frac{1}{s^2+0.2s+2}$$

Give an explanation of how dead-time affects Bode and Nyquist diagrams. Illustrate with several examples.

Give an explanation of bandwidth and illustrate with 3 examples of your choice. Include the Bode diagrams.

Sketch, using asymptotes, the Bode diagrams of the following transfer functions and verify them in MATLAB. How would the Bode plots change if G were modified as G → GK where K=(s+1)/(s+5) and K=(s+5)/(s+1):

$$\frac{100}{s+5};\quad \frac{20}{s+3};\quad \frac{10}{(s+1)(s+2)};\quad \frac{100}{(s+3)(s+4)};$$

$$\frac{10}{s(s+1)(s+2)};\quad \frac{0.4}{s(s+2)(s+5)};\quad \frac{0.2(s+1)}{(s+2)(s+3)};\quad \frac{5(s+2)(s+1)}{s^2(s+3)};$$

Common compensators (Lead and Lag) take the form below. Plot the bode gain and phase plots for K=1, a=1 and r=10 for each of these. What do you notice that is different about them?

$$K(s)=K\frac{s+a}{s+ra};\quad r>1 \qquad K(s)=K\frac{s+ra}{s+a};\quad r>1 \qquad G(s)=\frac{5}{s(s+2)^2}$$

- Sketch the Bode diagrams for other lead and lag compensators of your choice and explain the key attributes of each and how these might be used for design.
- By sketching the relevant Bode and Nyquist diagrams, demonstrate the effect of the lead and lag compensators above on a system G(s).
- What is a lead-lag compensator. Sketch the Bode diagram for one of these and explain the key attributes and how these might be used for design.

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1. The open-loop transfer function of a unity-feedback system is given by G(s) below. Sketch the Nyquist.
2. A chemical reactor plant can be modelled as a unity-feedback system whose open-loop transfer function is given by H(s) below. Sketch the Nyquist plot.

3. The open-loop transfer function of a unity feedback system is given by L(s) below. Sketch the Nyquist diagram.
4. What is the impact on the 3 Nyquist plots above of adding Lead or Lag compensators K=(s+1)/(s+5) and K=(s+5)/(s+1)?

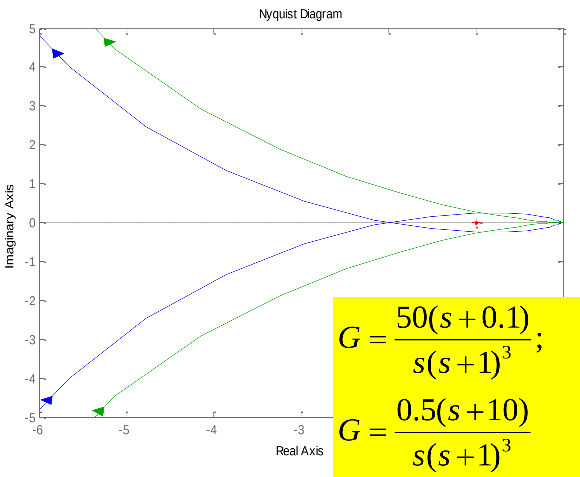
$$G(s)=\frac{10}{s(s+10)^2};\quad H(s)=\frac{80}{s(s+4)(s+10)};\quad L(s)=\frac{K}{s(1+0.5s)(1+0.1s)}$$

Analyse, with Nyquist, the closed-loop stability of the following when connected with unity negative feedback. Also investigate the impact of changes in gain. You could use MATLAB to generate the Nyquist diagrams.

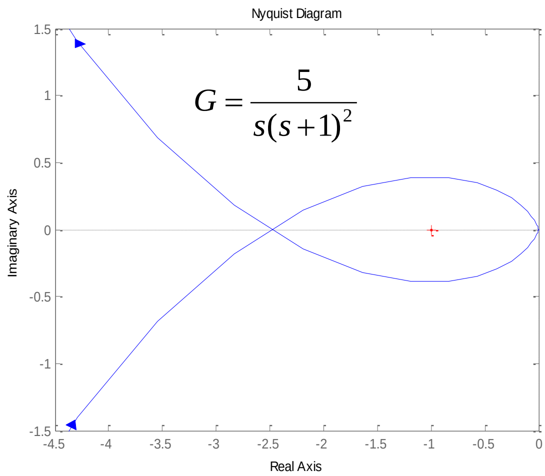
$$G1=\frac{3(s+1)}{s^2(s+0.1)};\quad G2=\frac{3(s+0.1)}{s^2(s+1)};\quad G3=\frac{s+0.1}{(s-1)(s+0.2)};\quad G4=\frac{s+3}{s(s+1)(s+2)(s+5)};$$

$$G5=\frac{14s+60}{s(s+1)(s+2)(s+5)};\quad G6=\frac{s+4}{s^2(s+1)(s+2)};\quad G7=\frac{400(s+4)(s+20)}{s^2(s+1)(s+100)};\quad G8=\frac{40(s+2)}{(s-1)(s+10)(s+4)}$$

Analyse, with Nyquist, the closed-loop stability of the following.



$$G=\frac{50(s+0.1)}{s(s+1)^3};\quad G=\frac{0.5(s+10)}{s(s+1)^3}$$

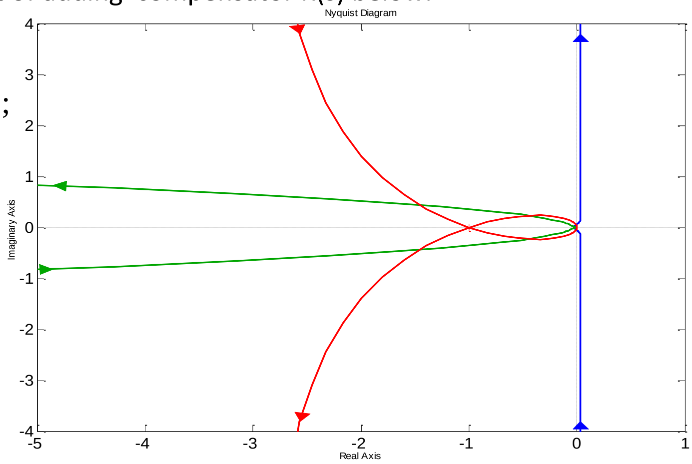


$$G=\frac{5}{s(s+1)^2}$$

Match the systems to the Nyquist plots (show your working) and hence comment on closed-loop stability with K=1 and the likely impact of adding compensator K(s) below.

$$G1=\frac{s+1}{s(s+4)(s+3)};\quad G2=\frac{s+1}{s^2(s+4)};$$

$$G3=\frac{s+2}{s(s-1)};\quad K(s)=k\frac{s+4}{s+1}$$



The open-loop transfer function of a unity-feedback system is given by G(s). Sketch the closed Nyquist plot and then verify that $\angle$G=-180 when w=10 and hence that the system is unstable in the closed-loop when K>2. Use the Nyquist criterion to determine how many RHP closed-loop poles the closed-loop system has when K=3? (Check your answer with root-loci.) ?

$$G(s) = \frac{10K}{s(1+0.1s)^2}$$

Compare the stability of the following processes with different compensators.

i) G(s) = 1/s(s+1); K(s)=1 or K(s)=(s+10)/(s+1) or K(s) =s+1/(s+10)

ii) G(s) = 0.5/(s-1); K(s)=1 or K(s)=4(s+1)/s

Do your sketches by hand. (Again you may find that root-loci give some insight.)

A chemical reactor plant can be modelled as a unity-feedback system whose open-loop transfer function is given by M(s). Assuming T=0: (i) sketch a Nyquist plot and hence estimate the values of positive K such that the system is stable and (ii) compute the number of RHP poles for large positive K and for negative K and (iii) Use MATLAB to illustrate the impact of T=0.1 on stability.

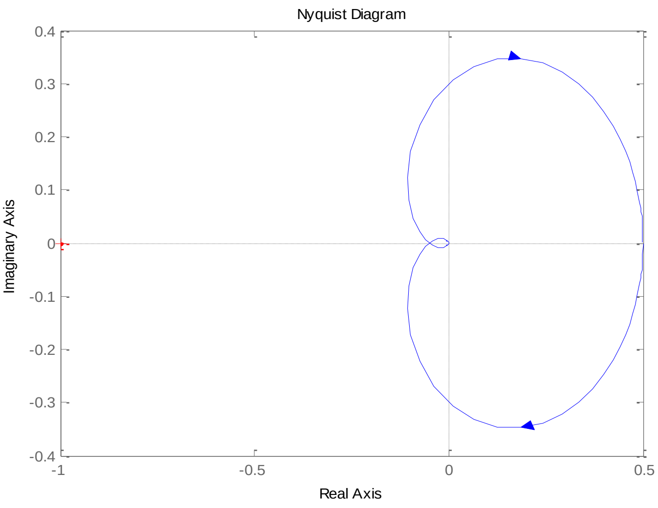
$$M(s) = \frac{80Ke^{-Ts}}{s(s+4)(s+10)}$$

Apply the Nyquist stability criteria to the following and hence determine which values of gain K give closed-loop stability.

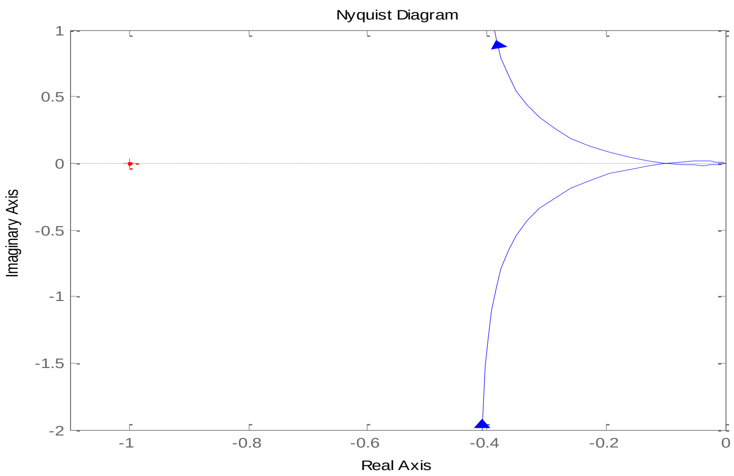
$$Q(s) = \frac{K(5s+3)}{s(s+1)(s+2)}; \quad Q(s) = \frac{K(10s+3)}{s^2(s+1)(s+2)}; \quad Q(s) = \frac{K(s+3)}{s^2(s+1)(s+2)}$$

$$G = \frac{K(s+0.2)}{(0.8-s)(6+s)}$$

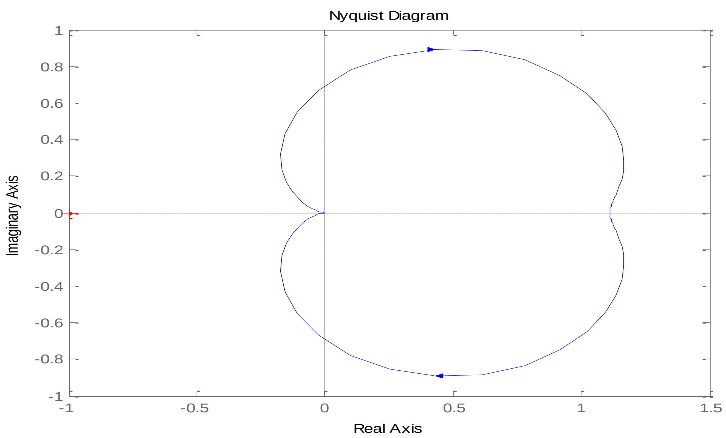
For each system following, determine whether it will be stable in the closed-loop with gains of K = 0.1, 1 and 10. The plots given are for K=1.



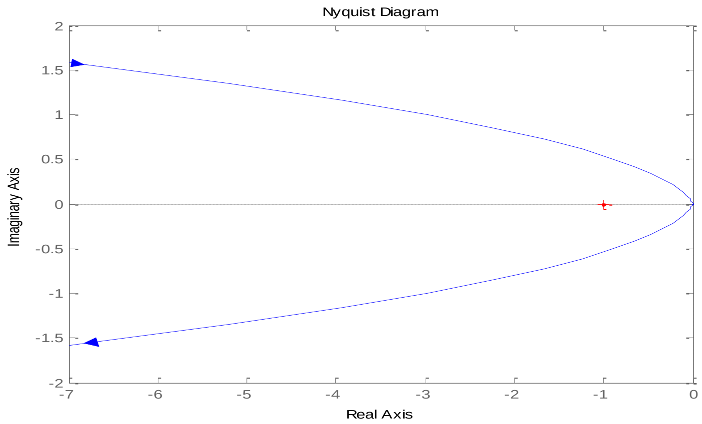
$$G(s) = \frac{3K}{(s+1)(s+2)(s+3)}$$



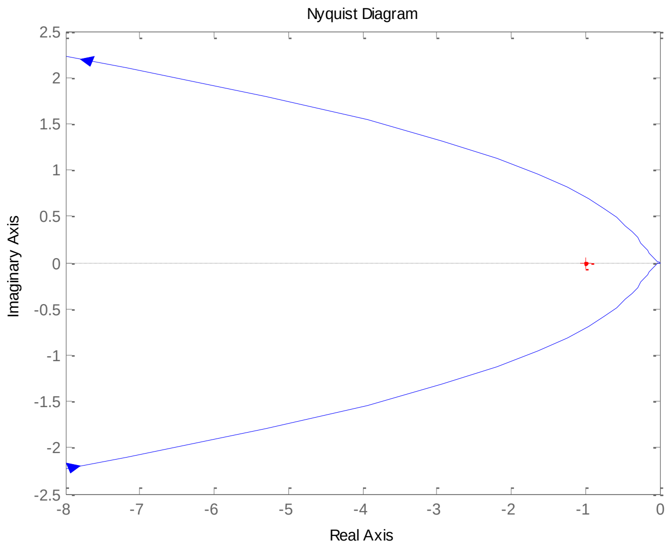
$$G(s) = \frac{3K}{s(s+2)(s+3)}$$



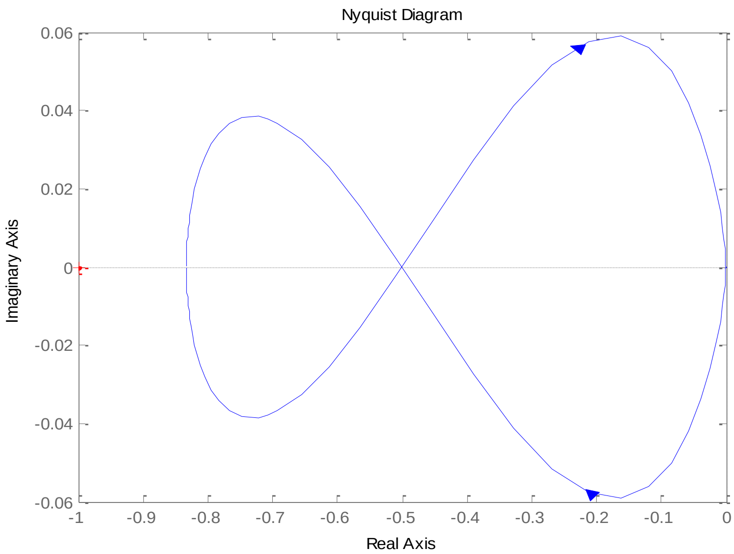
$$G(s) = \frac{100K(s+2)}{(s+10)(s+6)(s+3)}$$



$$G(s) = \frac{3K}{s^2(s+2)}$$



$$G(s) = \frac{5K(s+1)}{s^2(s+5)}$$



$$G(s) = \frac{5K}{(s-1)(s+2)(s+3)}$$

Sketch the Bode gain and phase asymptotes for the following system.

$$G(s) = 0.26\frac{3(s+9)}{s(s+2)(s+4)}$$

By computing some exact values of *G*(*jw*) at appropriate frequencies, improve your estimates of the Bode plots and hence improve your sketch. How did you choose your frequencies?

After first giving a clear definition of the gain and phase margins, show how you can use the Bode plots to estimate the gain and phase margins for *G*(*s*) . Mark the margins clearly on the Bode plots.

A lag compensator *K*(*s*) is used to improve the margins. Comment on the new margins.

$$K(s) = 0.26\frac{(s+0.09)}{(s+0.02)}$$

Sketch the Nyquist plot for *G*(*s*)*K*(*s*). What is the Nyquist stability criteria? Apply this to *G*(*s*)*K*(*s*) and hence comment on closed-loop stability with unity negative feedback.

Sketch the root-loci for *G*(*s*)K(s) and show on the diagram where you would like the closed-loop poles to be to get a balance between speed and damping.

What sort of closed-loop performance do you expect from your proposed lag design? Give your reasons!

The output is measured using a device that has a transfer function of $H(s) = \frac{4}{(s+4)}$

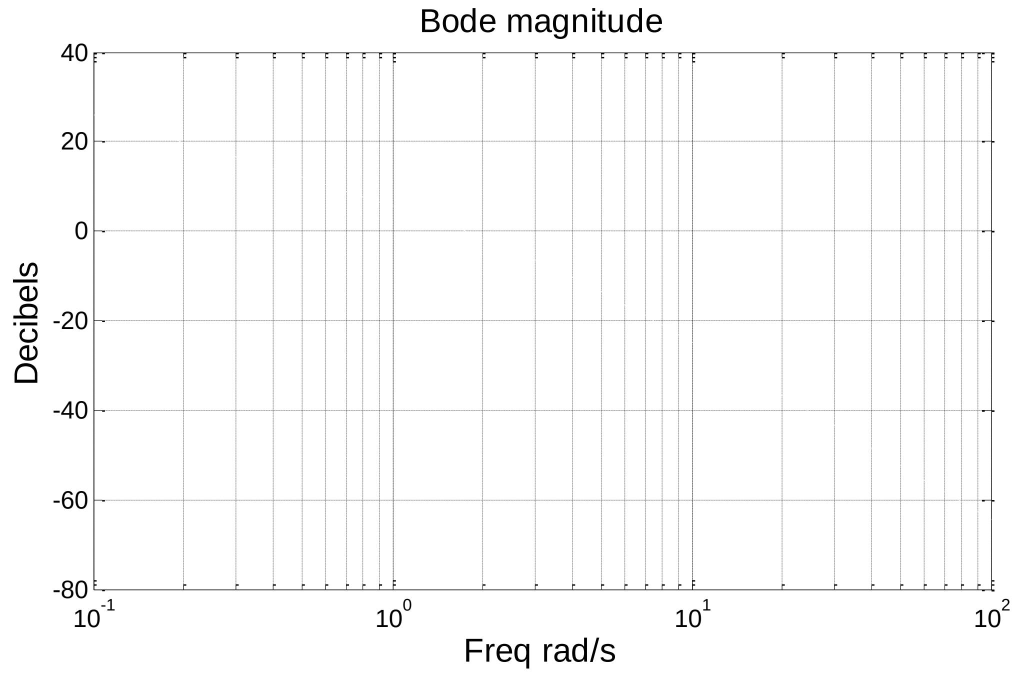
Sketch the new closed-loop block diagram where *H*(*s*) is now in the feedback path and define the new closed-loop transferences from set point to input and output.

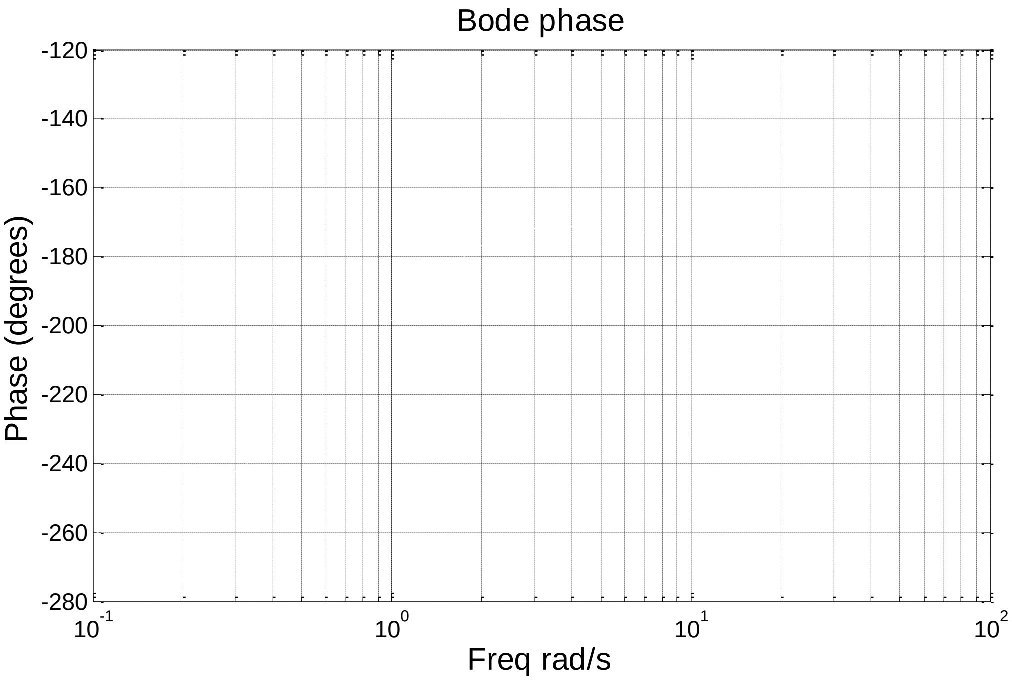
What is the impact of measurement dynamics on the expected behaviour of your loop? Use rough sketches of modified Bode plots, root-loci or otherwise to illustrate and/or justify your answer.

Using Bode, Nyquist and root-loci plots, analyse the closed-loop behaviour of a system G(s) with a scalar gain K=4 and hence discuss the expected impact of lead and lag compensators $K_1(s)$, $K_2(s)$.

$$G(s) = \frac{6(s+1)}{s(s-1)(s+3)}; \quad K_1 = \frac{4(s+2)}{s+5}; \quad K_2 = \frac{4(s+0.1)}{(s+0.04)}$$

Note that a frequency of √3 rad/s is the phase cross-over frequency. Bode graph paper is provided for convenience.





Sketch the Bode asymptotes and hence a more accurate Bode diagram for

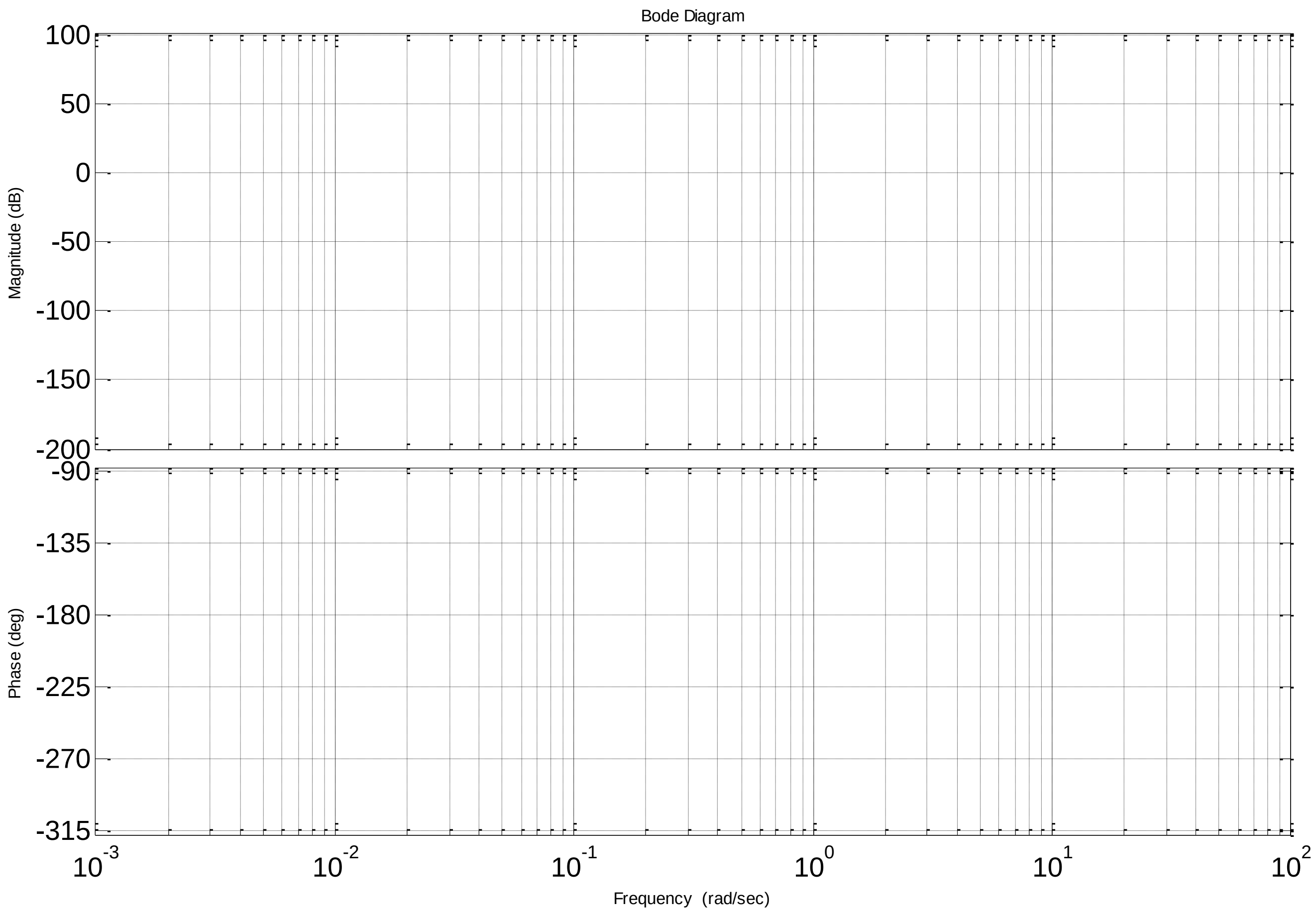
$$G(s) = \frac{10^{-3}(s+4)}{s(s+2)(s+0.1)^2}.$$

Use this Bode diagram to estimate the gain and phase margins, giving all your working and definitions clearly.

Choose a gain K to give good closed-loop response and justify your answer.

Which of the two compensators, *$H_1(s)$, $H_2(s)$* are likely to be more appropriate. Justify your answer with appropriate arguments, computations and sketches.

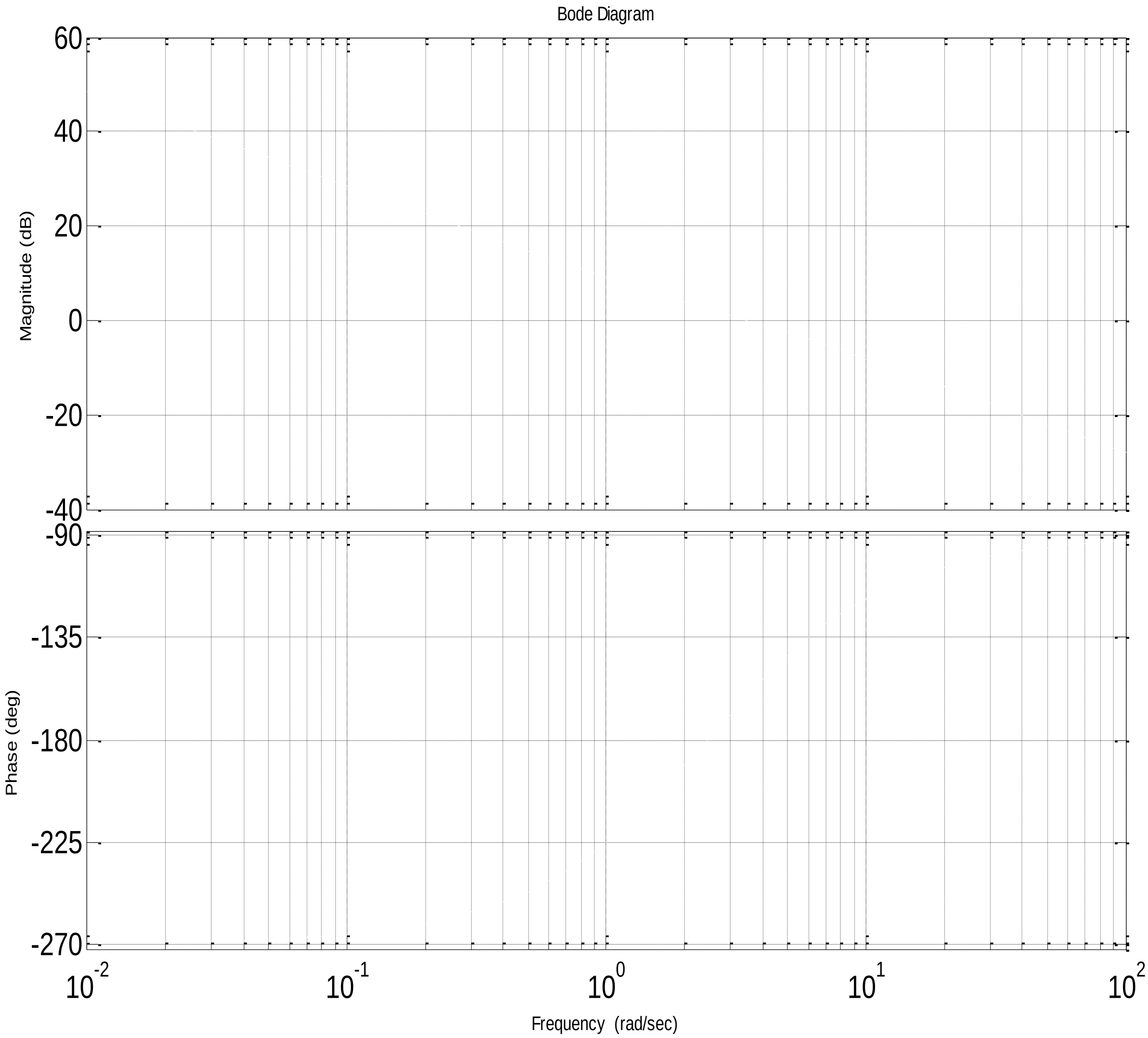
$$H_1(s) = 2\frac{s+0.06}{s+0.18}; \qquad H_2(s) = \frac{s+0.18}{3(s+0.06)}$$



You are given a system G(s). $G(s)=\dfrac{4(s+2)}{s(s-3)}$

By first forming the Bode, Nyquist and root-loci plots, analysis the expected behaviour in closed-loop with unity negative feedback. Hence given some comment on the likely efficacy of the following compensators.

$$K(s)=0.2;\quad K(s)=3;\quad K(s)=\frac{2(s+2)}{s+4}$$



By first generating sketches of the Bode, Nyquist and root-loci plots, give an analysis of the expected behaviour of the system *G(s)* with *K(s)* a constant and hence discuss what form of compensation could be helpful.

$$G(s) = \frac{4}{s(s+1)(s+3)}$$

- Sketch the NYQUIST diagram making all your working clear. Ensure you discuss the Nyquist stability criteria and show how it applies to this case.
- Sketch the root-loci making all your working clear.
- Using insights from the Bode, Nyquist and root-loci, give an analysis of the feedback loop with no compensation and a good constant compensator of your choice. Hence discuss what form of compensator maybe appropriate, giving your reasoning, and illustrate this on your Bode diagram.