Modelling and control summaries

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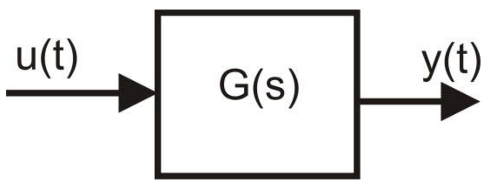
Bode 1: What is frequency response?

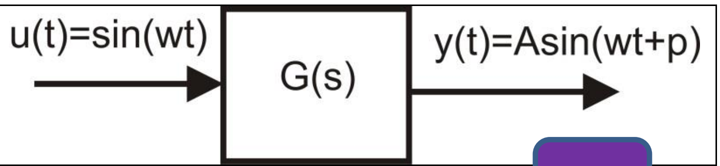
This brief summary assumes readers are familiar with the concept of feedback, transfer functions and block diagrams. For now this series assumes the open-loop arrangement below (u(t) the input, y(t) the output).



$$U(s)=L[u(t)]$$
$$Y(s)=L[y(t)]$$
$$Y(s)=G(s)U(s)$$

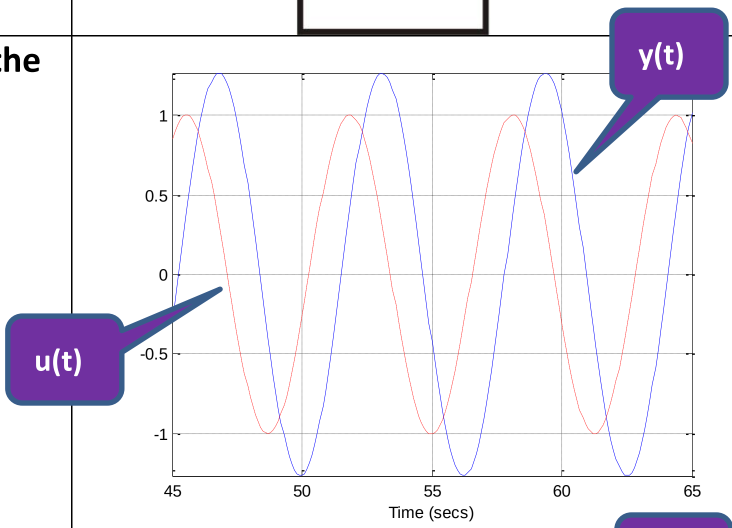
What is a frequency response?

This is the system response (or asymptotic behaviour) of the output when the input signal u(t) is a sinusoid.



The response is characterised by the variables A and p

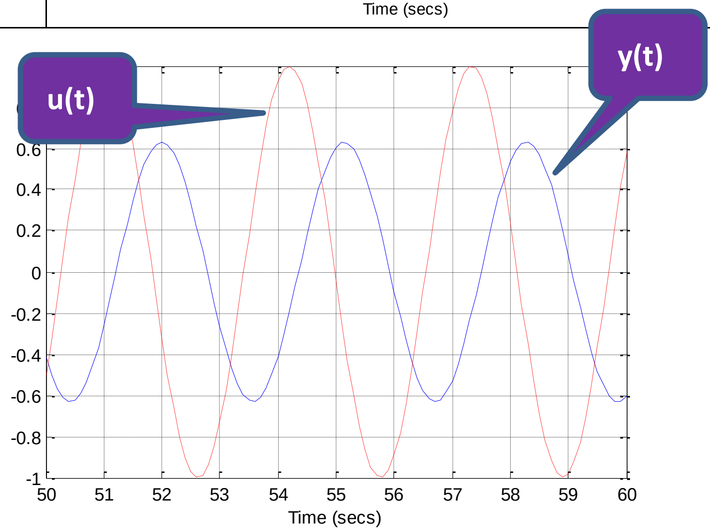
$$G=\frac{4}{s^2+3s+2}$$
$$u=\sin(t)$$
$$y=A\sin(t+p)$$
$$A\approx 1.2,\quad p=-\frac{\pi}{4}-\tan^{-1}\frac{1}{2}$$



Response is frequency dependent

Change the frequency and both A and p will change!

$$u=\sin 2t$$
$$y\approx 0.62\sin(2t-1.8)$$



Frequency response

As the frequency of the input changes:

- The amplitude *A* of the output changes.
- The phase shift *p* of the output changes.

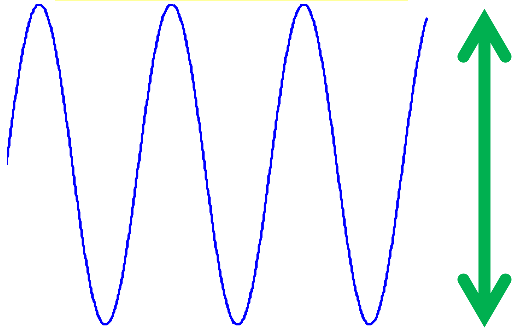
Frequency response is a description of how the amplitude and phase shift depend upon the frequency of the input, that is, how do the characteristics of the response, depend upon the frequency?

Phase shift is in radians – this corresponds to a time shift via the frequency of oscillation.

$$\sin(wt+\phi) \equiv \sin(wt+wt_s)$$
$$\phi = wt_s$$

Some systems include integrators and hence, even with a sinusoidal input, the output signal is not centred on zero. Hence, amplitude is half peak to peak. The following signal, has an amplitude of oscillation of A.

$$y = B + A\sin(wt+\phi)$$



Gain is the ratio of the output amplitude to the input amplitude.

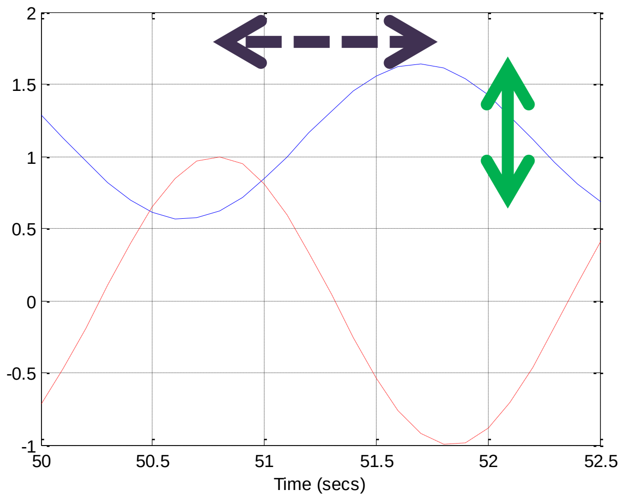
Question

Find the gain and phase for the following system for the frequency of 3rad/s. [that is u=sin(3t)]

The amplitude of oscillation is marked with the double green arrow; clearly A = (1.6-0.6)/2 = 0.5

The time shift (dotted double arrow) is t_s=50.8-51.7=-0.9s. Hence the phase shift is -2.7rad.

Remark: The mean output is not zero so the system contains an integrator.



SUMMARY: For sinusoidal inputs, the gain and phase shift of the output compared to the input depend upon frequency. This dependence is denoted as frequency response.

y = A sin (wt+φ)+B for a sinusoidal input *u=sin(wt)*.

A=A(w) is defined as the system gain and *φ=φ(w)* is defined as the system phase.

Frequency response is simply the information stored in *A=A(w)* and *φ=φ(w)*

QUESTIONS: Find the gain and phase for the following pairs of systems/inputs.

$$\left\{G(s)=\frac{4}{s+3}; u(t)=\sin(2t)\right\} \quad \left\{G(s)=\frac{0.2}{s+0.1}; u(t)=\sin(0.5t)\right\}$$