Modelling and control summaries
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**Bode 10: Sketching using asymptotes**

SUMMARY of Frequency response
We have established that Bode diagram comprises two plots (gain in dB, phase in degrees):
\[
gain = 20 \log_{10} |G(j\omega)| \quad \text{phase} = \angle G(j\omega)
\]

Asymptotes for a single factor of the form \( G = (s+a) \) [Figure does \( G = 1/(s+1) \)]

- **If \( w < a \)** approx. factor as \( (jw+a) \approx a \)
- **If \( w > a \)** approx. factor as \( (jw+a) \approx jw \)
- **If \( w = a \)** then have \( jw+a = ja+a \)

Gain is \( a \), phase is zero
Gain is \( w \) (or 20dB/dec), phase is 90
Gain is \( av2 \) (or 3dB up), phase is 45.

It is useful to consider the phase at a decade above and below the corner frequency as this gives an idea of distance from the asymptote.

\[
\begin{align*}
\omega &= a/(10) \quad \Rightarrow \quad \angle jw + a = \tan^{-1}(0.1) \approx 6^\circ; \\
\omega &= 10a \quad \Rightarrow \quad \angle jw + a = \tan^{-1}(10) \approx 84^\circ
\end{align*}
\]

**EXAMPLE 2** - Use asymptotic information for each factor in turn and then combine these together to form an overall sketch.

- \( w < 1 \) \( jw+1 \approx 1 \) \( jw+5 \approx 5 \) \( G(jw) \approx \frac{3}{5} \) (or -4dB)
- \( 1 < w < 5 \) \( jw+1 \approx jw \) \( jw+5 \approx 5 \) \( G(jw) \approx \frac{3}{5jw} \)
- \( w > 5 \) \( jw+1 \approx jw \) \( jw+5 \approx jw \) \( G(jw) \approx \frac{3}{3(jw)^2} \)
- \( w = \sqrt{5} \) \( \angle G(jw) = -90 \) \( |G(jw)| = \frac{1}{2\sqrt{5}} \) (or -13dB)

In plots, blue lines are asymptotes from these approximations, red lines are exact values.
EXAMPLE 3 - Use asymptotic information for each factor in turn and then combine these together to form an overall sketch.

\[ G = \frac{s + 1}{(s + 2)(s + 3)} \]

where the asymptotes are least representative.

Add some exact computations around the corner frequencies to ensure accuracy where the asymptotes are least representative.

\[ \angle G = \tan^{-1} \frac{w - \tan^{-1} \frac{w}{2} - \tan^{-1} \frac{w}{3}}{1}; \quad \angle G(j1) \approx 0, \angle G(j3) \approx -30 \]

SUMMARY

1. Usually the errors in the gain plot are relatively small compared to the accuracy at which a human can do a hand sketch, especially with ranges of 60-80 dB. Consequently it is often not worth embellishing gain asymptotes unless you are aware (e.g. double poles) that the error is likely to be significant.

2. With phases, 2-3 exact computations may be required near the corner frequencies to ensure the sketch is accurate enough.

3. Use a computer if you really need an accurate plot. The purpose of practising sketching is to gain insight and understanding which subsequently is useful for design.

4. In general, the plots only need to be accurate in the region -120 to -180 degrees, and thus one can focus numerical effort quite easily.