Modelling and control summaries

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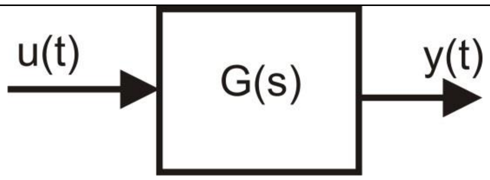
Bode 10: Sketching using asymptotes



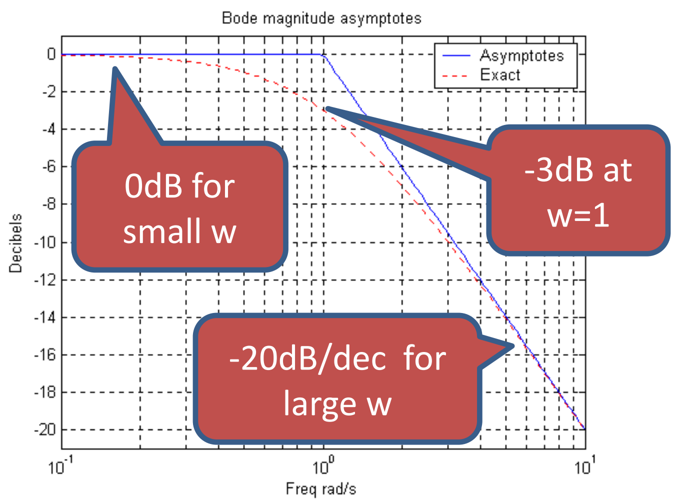
SUMMARY of Frequency response

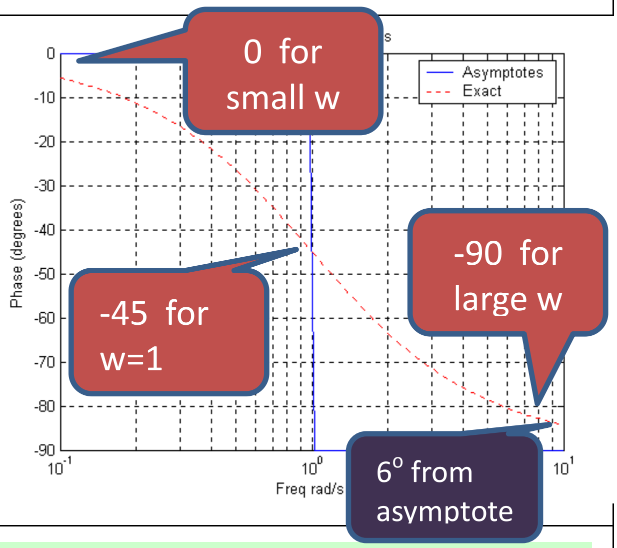
We have established that Bode diagram comprises two plots (gain in dB, phase in degrees):

$$gain = 20\log_{10}|G(jw)|; \quad phase = \angle G(jw)$$

Asymptotes for a single factor of the form G =(s+a) **[Figure does G=1/(s+1)]**

If w<a approx. factor as (jw+a)=a	**Gain is a, phase is zero**
If w>a approx. factor as (jw+a)=jw	**Gain is w (or 20dB/dec), phase is 90**
If w=a then have jw+a=ja+a	**Gain is a√2 (or 3dB up), phase is 45.**





It is useful to consider the phase at a decade above and below the corner frequency as this gives an idea of distance from the asymptote.

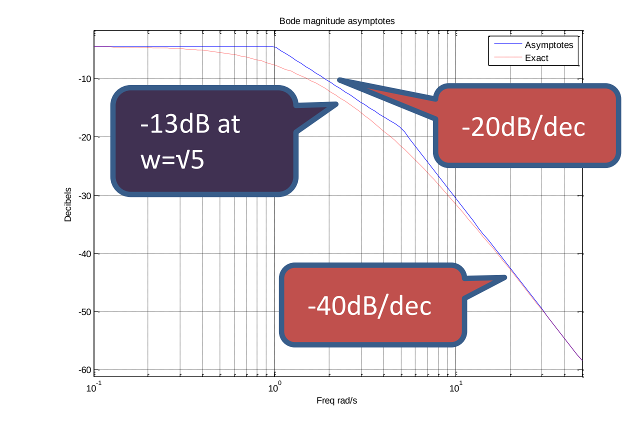
$$\omega = a/(10) \quad \Rightarrow \quad \angle jw + a = \tan^{-1}(0.1) \approx 6^o;$$
$$\omega = 10a \quad \Rightarrow \quad \angle jw + a = \tan^{-1}(10) \approx 84^o$$

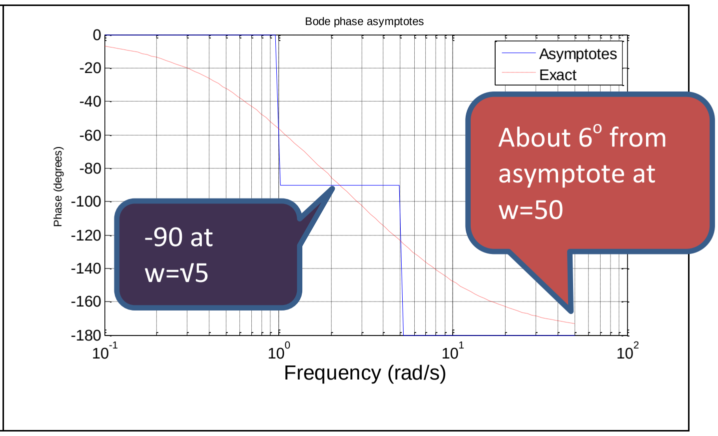
EXAMPLE 2 - Use asymptotic information for each factor in turn and then combine these together to form an overall sketch.

$$G = \frac{3}{(s+1)(s+5)}$$

$w<1$	$jw+1 \approx 1$	$jw+5 \approx 5$	$G(jw) \approx \frac{3}{5} (or -4dB)$
$1<w<5$	$jw+1 \approx jw$	$jw+5 \approx 5$	$G(jw) \approx \frac{3}{5jw}$ — -20dB/dec and -90^o
$w>5$	$jw+1 \approx jw$	$jw+5 \approx jw$	$G(jw) \approx \frac{3}{(jw)^2}$ — -40dB/dec and -180^o
$w=\sqrt{5}$	$\angle G(jw) = -90$	$\lvert G(jw)\rvert = \frac{1}{2\sqrt{5}}$	$(or \quad -13dB)$

In plots, blue lines are asymptotes from these approximations, red lines are exact values.





EXAMPLE 3 - Use asymptotic information for each factor in turn and then combine these together to form an overall sketch.

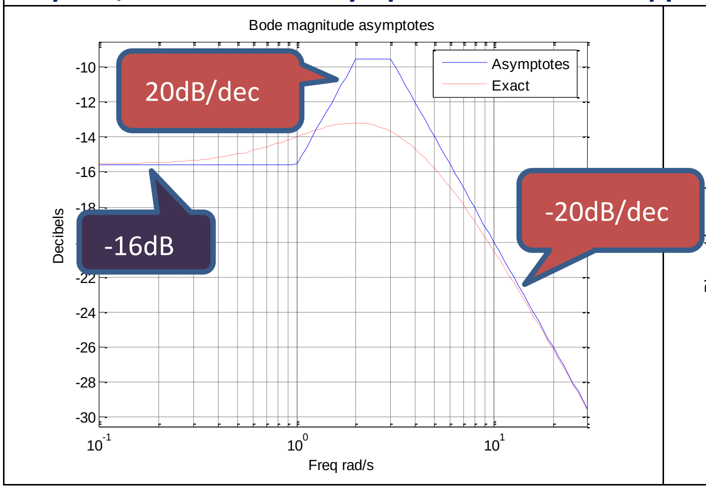
$$G=\frac{s+1}{(s+2)(s+3)}$$

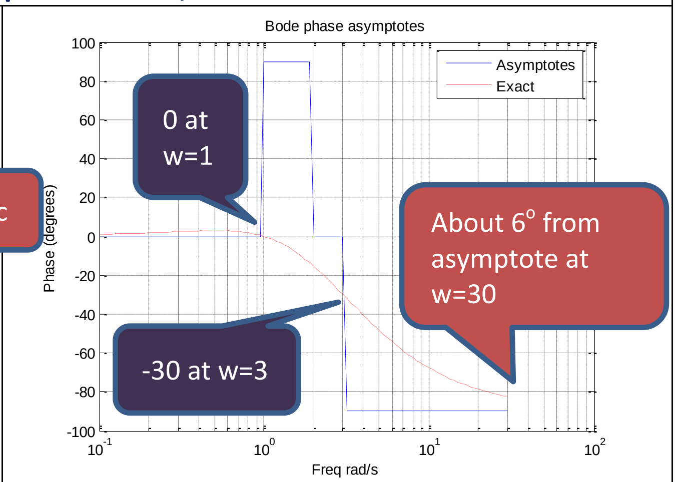
$$\begin{array}{lllll} w<1 & jw+1\approx 1 & jw+2\approx 2 & jw+3\approx 3 & G(jw)\approx 1/6\ (or\ -16dB) \\ 1<w<2 & jw+1\approx jw & jw+2\approx 2 & jw+3\approx 3 & G(jw)\approx jw/6 \\ 2<w<3 & jw+1\approx jw & jw+2\approx jw & jw+3\approx 3 & G(jw)\approx 1/3 \\ w>3 & jw+1\approx jw & jw+2\approx jw & jw+3\approx jw & G(jw)\approx 1/jw \end{array}$$

Add some exact computations around the corner frequencies to ensure accuracy where the asymptotes are least representative.

$$\angle G=\tan^{-1}w-\tan^{-1}\frac{w}{2}-\tan^{-1}\frac{w}{3};\quad \angle G(j1)\approx 0,\ \angle G(j3)\approx -30$$

In plots, blue lines are asymptotes from these approximations, red lines are exact values.





SUMMARY

1. Usually the errors in the **gain plot** are relatively small compared to the accuracy at which a human can do a hand sketch, especially with ranges of 60-80 dB. Consequently it is often not worth embellishing gain asymptotes unless you are aware (e.g. double poles) that the error is likely to be significant.
2. With phases, 2-3 exact computations may be required near the corner frequencies to ensure the sketch is accurate enough.
3. **Use a computer if you really need an accurate plot**. The purpose of practising sketching is to gain insight and understanding which subsequently is useful for design.
4. **In general, the plots only need to be accurate in the region -120 to -180 degrees, and thus one can focus numerical effort quite easily.**