Modelling and control summaries

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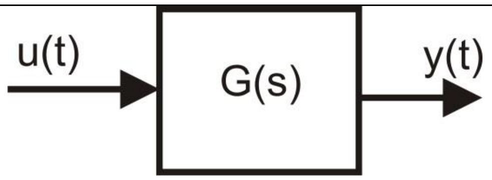
Bode 12: Lag compensators



SUMMARY of Frequency response

We have established that Bode diagram comprises two plots (gain in dB, phase in degrees):

$$gain = 20\log_{10}|G(jw)|; \quad phase = \angle G(jw)$$

Definition of lag compensator and key attributes

$$K\frac{s+\beta a}{s+a}; \quad 1 \le \beta \le 10$$

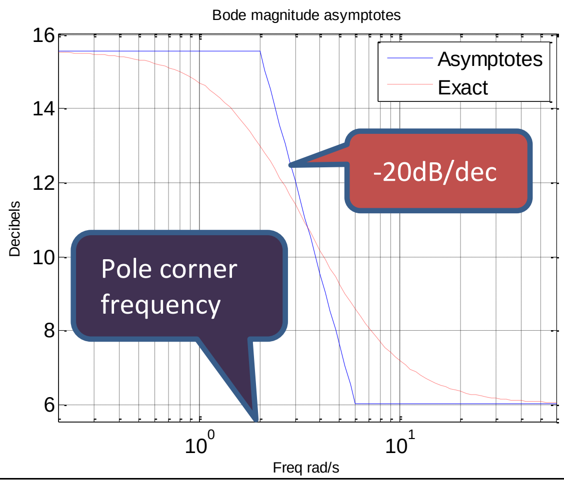
One pole and one zero.
Zero to the left of pole in argand diagram.
Three parameters (K,a and β)

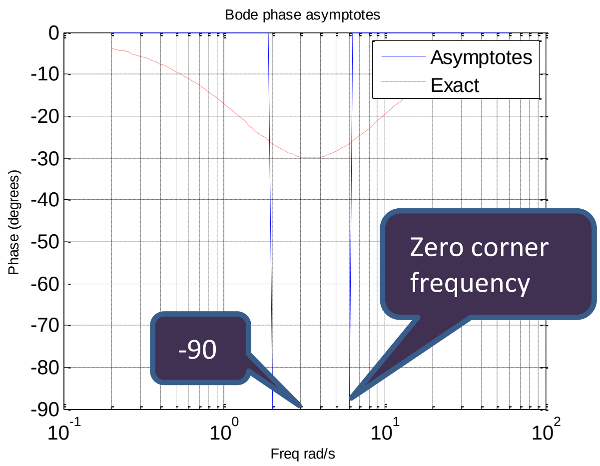
KEY Observations for Bode diagram: As zero corner frequency is largest,

1. gain asymptote goes [0, -20db/dec,0]
2. phase asymptote goes [0 -90 0]

Take the system K(s):

$$K(s) = 2\frac{s+6}{s+2}$$



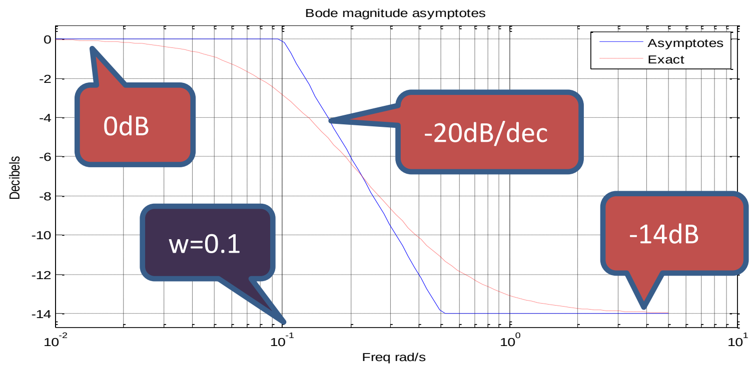
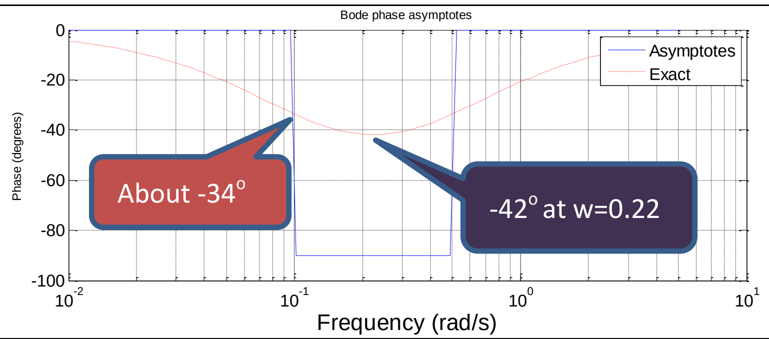


1. Gain is always reducing and thus is maximum at low frequency and minimum at high frequency – this makes LAG at low gain compensator as it reduces gain in the mid-frequency range compared to steady-state.
2. Phase is always negative and is most negative in between the two corner frequencies. However phase is approximately zero at both high and low frequencies.

KEY QUESTIONS

1. How does the phase characteristic depend on the zero/pole ratio?
2. How does the gain characteristic depend upon the pole/zero ratio?
3. How do I choose the pole/zero positions?

KEY ATTRIBUTES (Gain and phase)	
$M = K\dfrac{s+\beta a}{s+a}$	1. LOW FREQUENCY GAIN IS Kβ 2. HIGH FREQUENCY GAIN IS K 3. Ratio of low to high frequency gains is β. A typical lag design focuses on the choice of β, not K, that is ratio of low to high frequency gain. K is determined by requirements which could be considered separately to the lag design procedure.
$\angle M = \tan^{-1}\dfrac{w}{\beta a} - \tan^{-1}\dfrac{w}{a}$	The phase is always negative as β>1. The phase at corner frequencies w=a and w= βa must be the same (from symmetry). **Also, depends solely on β!** $\tan^{-1}\dfrac{a}{\beta a} - \tan^{-1}\dfrac{a}{a} = \tan^{-1}\dfrac{\beta a}{\beta a} - \tan^{-1}\dfrac{\beta a}{a} = \tan^{-1}\dfrac{1}{\beta} - 45$
The largest phase occurs at the geometric mean w_m of the corner frequencies and depends solely on β. **Often ignored in design as phase dip is not near critical frequency range.**	$w_m = a\sqrt{\beta}$ $\tan^{-1}\dfrac{a\sqrt{\beta}}{\beta a} - \tan^{-1}\dfrac{a\sqrt{\beta}}{a} = \tan^{-1}\dfrac{1}{\sqrt{\beta}} - \tan^{-1}\sqrt{\beta}$ $= \tan^{-1}\left(\dfrac{\dfrac{1}{\sqrt{\beta}} - \sqrt{\beta}}{1 + \dfrac{1}{\sqrt{\beta}}\sqrt{\beta}}\right) = \tan^{-1}\left(\dfrac{\dfrac{1}{\sqrt{\beta}} - \sqrt{\beta}}{2}\right)$

EXAMPLE	
$0.2\dfrac{s+0.5}{s+0.1}$; $\beta = 5, K = 0.2, a = 0.1$ $w_m = 0.1\sqrt{5} \approx 0.22$	
Low Freq gain is: 1 or 0dB **High freq gain is: 0.2 or -14dB** **Phase peak is -42°** **At corner frequencies the phase is -34°**	

<u>REMARK</u>: In design it is the gain characteristic which is used and the phase characteristic is ignored by placing the corner frequencies a decade below the gain cross-over frequency.