

Modelling and control summaries



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Bode 13: The impact of lag compensators

Definition of lag compensator and key attributes

$$K \frac{s + \beta a}{s + a}; \quad 1 \leq \beta \leq 10$$

$$\phi = \tan^{-1} \left(\left(\frac{1}{\sqrt{\beta}} - \sqrt{\beta} \right) / 2 \right)$$

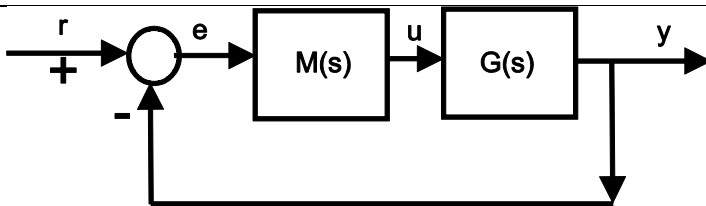
Low frequency gain is $K\beta$

High frequency gain is K

At geometric mean $\omega_m = a\sqrt{\beta}$ of corner frequencies, phase is ϕ .

At corner frequencies phase = $\tan^{-1}(1/\beta) - 45^\circ$:

IMPACT of compensation



Phase(GM)=Phase(G)+Phase(M)

$20\log_{10}|GM| = 20\log_{10}|G| + 20\log_{10}|M|$

Multiplying factors leads to addition in BOTH bode plots!

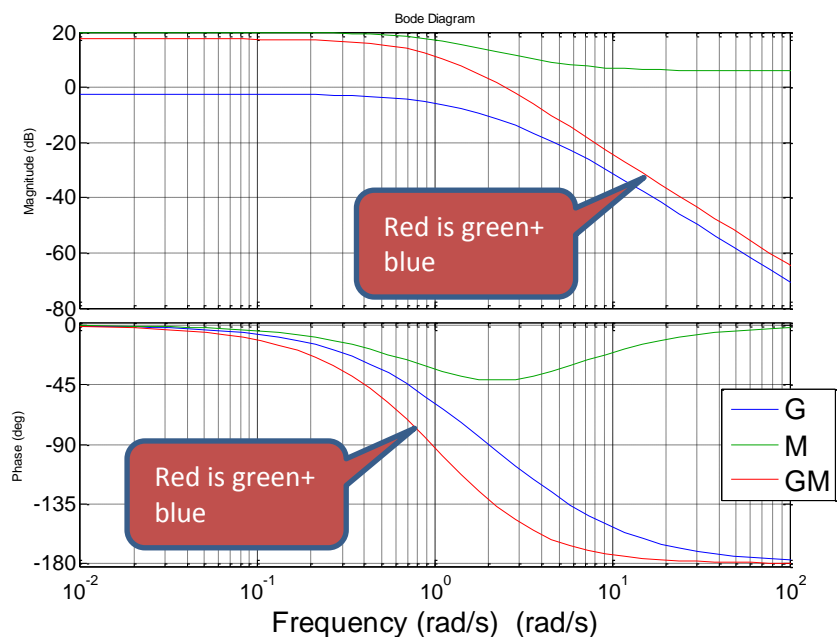
SUMMARY

- First plot the Bode diagram for $G(s)$ and the Bode diagram for $M(s)$
- The Bode diagram for $G(s)M(s)$ is determined by adding the two underlying plots.
- **ALTERNATIVELY**, view GM as a shifted plot from G , where the key gain and phase shifts are taken from M .

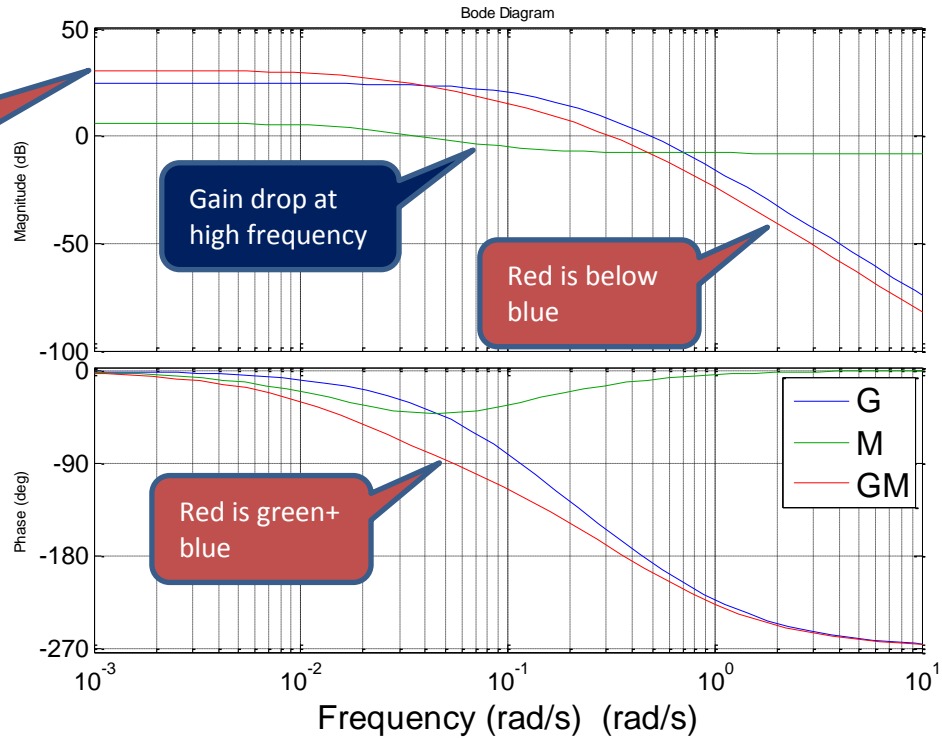
EXAMPLE

In the gain plot, $M(s)$ is around 20dB and thus one can see the blue plot ($G(s)$) is shifted up by about 20dB to give the compensated plot (in red).

IN the phase plot, one can see that where the phase of $M(s)$ is large (around 45° at $\omega=2$), the blue plot is shifted down by this amount to give the red plot



EXAMPLE 2



KEY OBSERVATION

1. A lag compensator moves the gain down more at high frequency relative to low frequency. For instance, in example 2 one can see red above blue at low frequency, but red below blue at high frequency.
2. This gain drop is the key attribute to be used later.
3. In design the phase characteristic is ignored by placing the corner frequencies a decade below the gain cross-over frequency.