Modelling and control summaries

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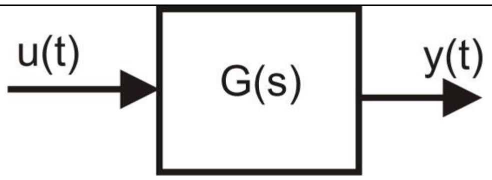
Bode 14: Lead compensators



SUMMARY of Frequency response

We have established that Bode diagram comprises two plots (gain in dB, phase in degrees):

$$gain = 20\log_{10}|G(jw)|; \quad phase = \angle G(jw)$$

Definition of lead compensator and key attributes

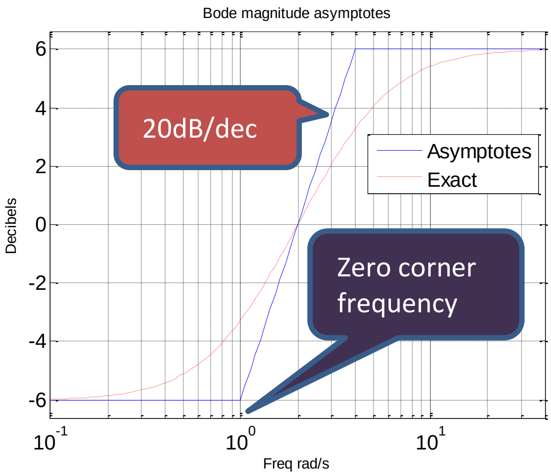
$K\dfrac{s+a}{s+\beta a}; \quad 1 \le \beta \le 10$	**One pole and one zero.** **Zero to the right of pole in argand diagram.** **Three parameters (K,a and β)**

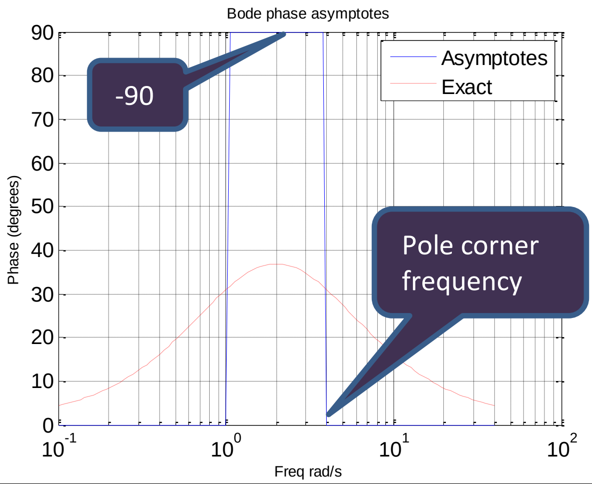
KEY Observations for Bode diagram: As zero corner frequency is largest,

1. gain asymptote goes [0, 20db/dec,0]
2. phase asymptote goes [0 90 0]

Take the system K(s):

$$K(s) = 2\frac{s+1}{s+4}$$





1. Gain is always increasing and thus is maximum at high frequency and minimum at low frequency – this makes LEAD a high gain compensator as it increases gain in the mid-frequency range compared to steady-state.
2. Phase is always positive and is most positive in between the two corner frequencies. However phase is approximately zero at both high and low frequencies.

KEY QUESTIONS

1. How does the phase characteristic depend on the zero/pole ratio?
2. How does the gain characteristic depend upon the pole/zero ratio?
3. How do I choose the pole/zero positions?

KEY ATTRIBUTES (Gain and phase)

$$M = K\frac{s+a}{s+\beta a}$$

1. HIGH FREQUENCY GAIN IS K/β
2. LOW FREQUENCY GAIN IS K
3. Ratio of high to low frequency gains is β.

A typical lead design focuses on the choice of β and a.
K is determined afterwards.

$$\angle M = \tan^{-1}\frac{w}{a} - \tan^{-1}\frac{w}{a\beta}$$

The phase is always positive as β>1.
The phase at corner frequencies w=a and w= βa must be the same (from symmetry). **Also, depends solely on β!**

$$\tan^{-1}\frac{a}{a} - \tan^{-1}\frac{a}{a\beta} = \tan^{-1}\frac{\beta a}{a} - \tan^{-1}\frac{\beta a}{a\beta} = 45 - \tan^{-1}\frac{1}{\beta}$$

The largest phase occurs at the geometric mean w_m of the corner frequencies and depends solely on β.

This value is key in design. Moreover w_m is expected to be close to the gain cross over frequency.

$$w_m = a\sqrt{\beta}$$

$$\tan^{-1}\frac{a\sqrt{\beta}}{a} - \tan^{-1}\frac{a\sqrt{\beta}}{a\beta} = \tan^{-1}\sqrt{\beta} - \tan^{-1}\frac{1}{\sqrt{\beta}}$$

$$= \tan^{-1}\left(\frac{\sqrt{\beta} - \frac{1}{\sqrt{\beta}}}{1 + \frac{1}{\sqrt{\beta}}\sqrt{\beta}}\right) = \tan^{-1}\left(\frac{\sqrt{\beta} - \frac{1}{\sqrt{\beta}}}{2}\right)$$

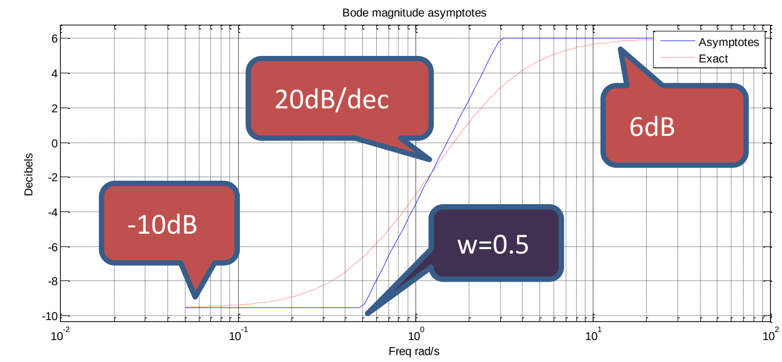
β	2	3	4	6	8	10
Phase peak	19	30	37	46	51	55

EXAMPLE

$$2\frac{s+0.5}{s+3};$$

$$\beta = 6, K = 2, a = 0.5$$

$$w_m = 0.5\sqrt{6} \approx 1.2$$



Low Freq gain is: 2 or 6dB
High freq gain is: 0.33 or approx. -10dB

Phase peak is 46°
At corner frequencies the phase is 36°