Modelling and control summaries

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Bode 15: The impact of lead compensators

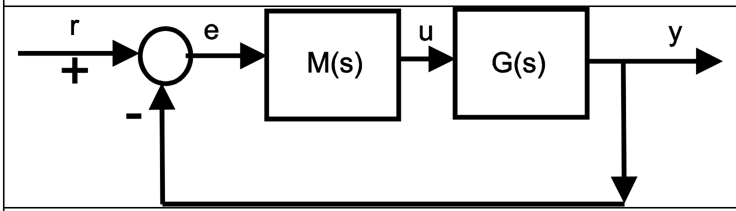
Definition of lag compensator and key attributes

$$K\frac{s+a}{s+\beta a};\quad 1\leq\beta\leq 10$$

$$\phi=\tan^{-1}\left((\sqrt{\beta}-\frac{1}{\sqrt{\beta}})/2\right)$$

Low frequency gain is K/β
High frequency gain is K
At geometric mean $w_m=a\sqrt{\beta}$ of corner frequencies, phase is φ.
At corner frequencies phase $=45-\tan^{-1}(1/\beta)$:

IMPACT of compensation



Phase(GM)=Phase(G)+Phase(M)
$20\log_{10}|GM|=20\log_{10}|G|+20\log_{10}|M|$

Multiplying factors leads to addition in BOTH bode plots!

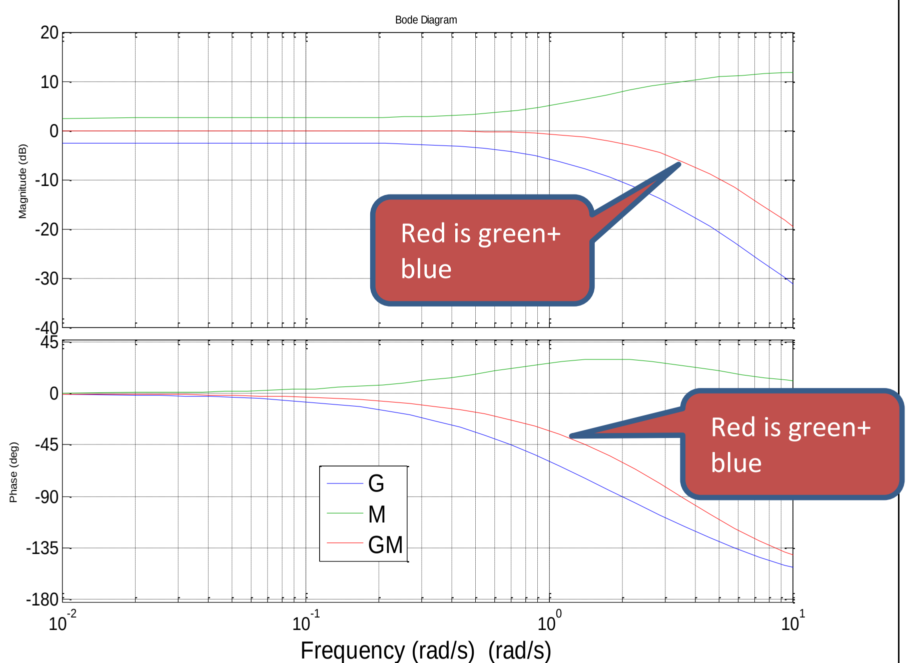
SUMMARY

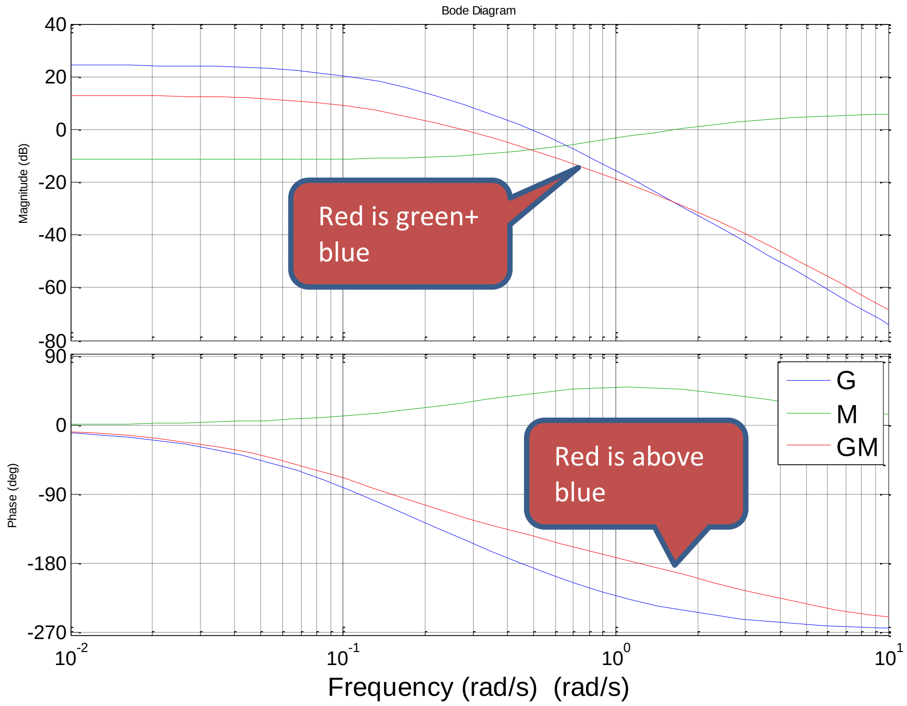
- First plot the Bode diagram for G(s) and the Bode diagram for M(s)
- The Bode diagram for G(s)M(s) is determined by adding the two underlying plots.
- **ALTERNATIVELY**, view GM as a shifted plot from G, where the key gain and phase shifts are taken from M.

EXAMPLE

In the gain plot, M(s) is around 3-12dB and thus one can see the blue plot (G(s)) is shifted up by about 3dB at low frequency and 12dB at high frequency.

In the phase plot, one can see that where the phase of M(s) is large (around 30° at w=2), the blue plot is shifted up by this amount to give the red plot



EXAMPLE 2	

KEY OBSERVATION

1. **A lead compensator moves the gain down more at low frequency relative to high frequency. For instance, in example 2 one can see red below blue at low frequency, but red above blue at high frequency.**
2. **This gain drop is the an attribute that in fact is ignored in design.**
3. **In design the phase characteristic is the core attribute to be used. Specifically it is noted that the red phase plot is always above the blue plot, that is one can lift the phase. The maximum phase shift is at the geometric mean of the corner frequencies and this is ideally near to the gain cross-over frequency.**