

Modelling and control summaries



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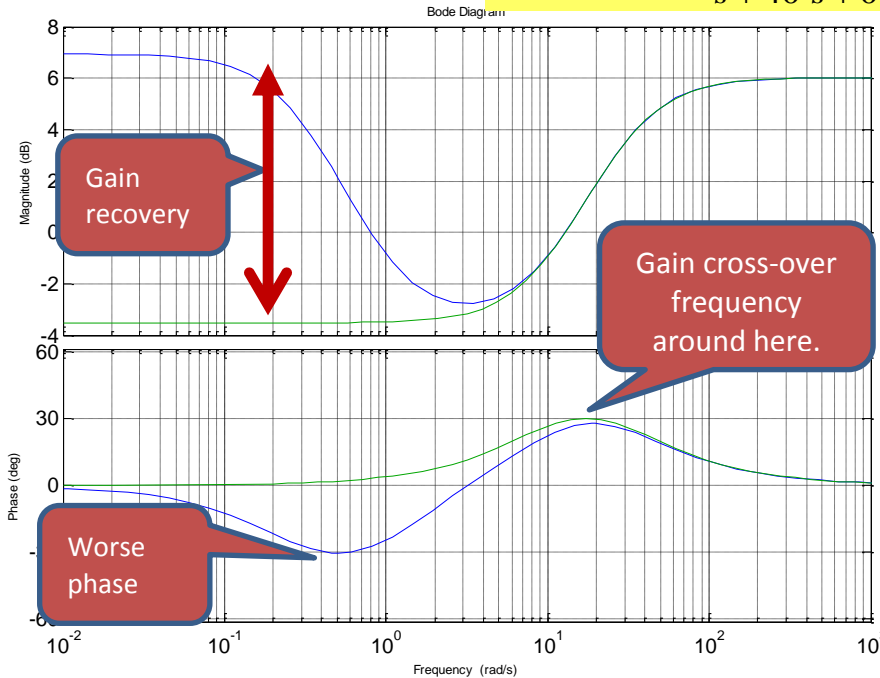
Bode 16: Lead-lag compensators

<p>SUMMARY of lead and lag It is noted that lead and lag have opposite attributes and thus play a different role in compensator design.</p>	<ol style="list-style-type: none"> LAG gives relatively high gain at low frequency, or conversely, relatively low gain at high frequency. However, this comes with negative phase around the corner frequencies. LEAD has relatively high gain at high frequencies and also positive phase around the corner frequencies.
<p>It is logical to combine lead and lag and thus exploit the useful attributes of both.</p>	

<p>Example of a lead-lag compensator and key attributes</p>	
$6 \frac{s+1}{s+4} \frac{s+0.2}{s+0.04}$	<p>Pole/zero of lead are always about a decade above the pole/zero of the lag. HERE lead has pole/zero of (-4,-1). Lag has pole/zero of (-0.04,-0.2)</p>
<p>The lead causes a loss of low frequency gain as shown here in the middle of the plot.</p> <p>Adding a lag helps regain low frequency gain.</p>	<p>Bode magnitude asymptotes</p> <p>The plot shows magnitude in Decibels (y-axis, 4 to 18) versus Frequency in rad/s (x-axis, log scale from 10⁻³ to 10²). A blue line represents the asymptotes, and a red line represents the exact magnitude. Key features are labeled: 'Gain recovery' at low frequencies, 'Low gain' in the middle, 'lag corner frequency' at approximately 0.04 rad/s, and 'lead corner frequency' at approximately 4 rad/s.</p>
<p>The lead has a positive phase characteristic which can be used to improve phase margins.</p> <p>The negative phase from the lag component is well to the left and hence not near the gain cross over frequency.</p>	<p>Bode phase asymptotes</p> <p>The plot shows Phase in degrees (y-axis, -100 to 100) versus Frequency in rad/s (x-axis, log scale from 10⁻³ to 10²). A blue line represents the asymptotes, and a red line represents the exact phase. Key features are labeled: 'Negative phase' at approximately 0.04 rad/s and 'Positive phase' at approximately 4 rad/s.</p>

COMPARE LEAD-LAG TO LEAD ONLY

$$\text{Lead_lag} = 2 \frac{s+10}{s+40} \frac{s+1}{s+0.3}; \quad \text{Lead} = 2 \frac{s+10}{s+40}$$



Just lead has worse low frequency gain but slightly better phase.

However, phase characteristic well to left of gain cross-over frequency.

KEY ATTRIBUTES (Gain and phase)

$$K \frac{s+a}{s+\beta a} \frac{s+\gamma b}{s+b}; \quad 1 \leq \beta \leq 10; \quad 1 \leq \gamma \leq 10; \quad a > b$$

1. Steady state gain is $K\gamma/\beta$, mid frequency gain is K/β and high frequency gain is K . Lead only would have a steady-state gain of K/β which is smaller.
2. Phase is zero at high and low frequency.
3. Phase is positive around corner frequencies of lead component and negative around corner frequencies of lag component.
4. Lead corner frequencies larger than lag corner frequencies.