Modelling and control summaries

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Bode 16: Lead-lag compensators

SUMMARY of lead and lag It is noted that lead and lag have opposite attributes and thus play a different role in compensator design.	1. LAG gives relatively high gain at low frequency, or conversely, relatively low gain at high frequency. However, this comes with negative phase around the corner frequencies. 2. LEAD has relatively high gain at high frequencies and also positive phase around the corner frequencies.
It is logical to combine lead and lag and thus exploit the useful attributes of both.	

Example of a lead-lag compensator and key attributes

$$6\frac{s+1}{s+4}\frac{s+0.2}{s+0.04}$$

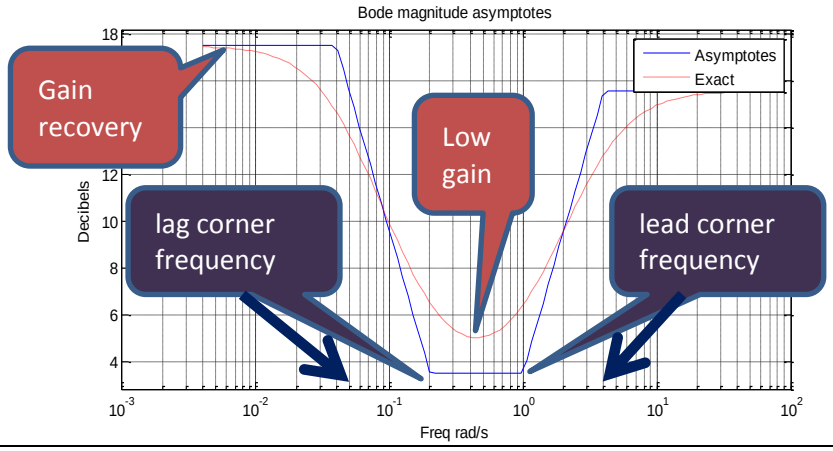
Pole/zero of lead are always about a decade above the pole/zero of the lag.

HERE lead has pole/zero of (-4,-1).

Lag has pole/zero of (-0.04,-0.2)

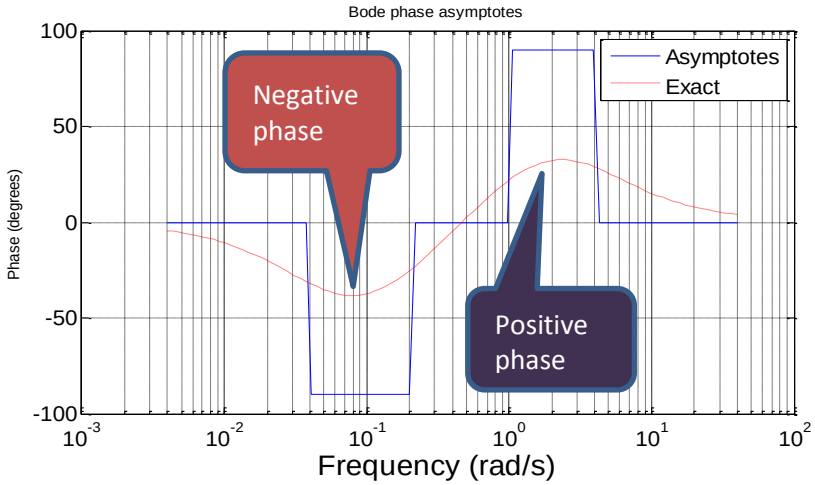
The lead causes a loss of low frequency gain as shown here in the middle of the plot.

Adding a lag helps regain low frequency gain.



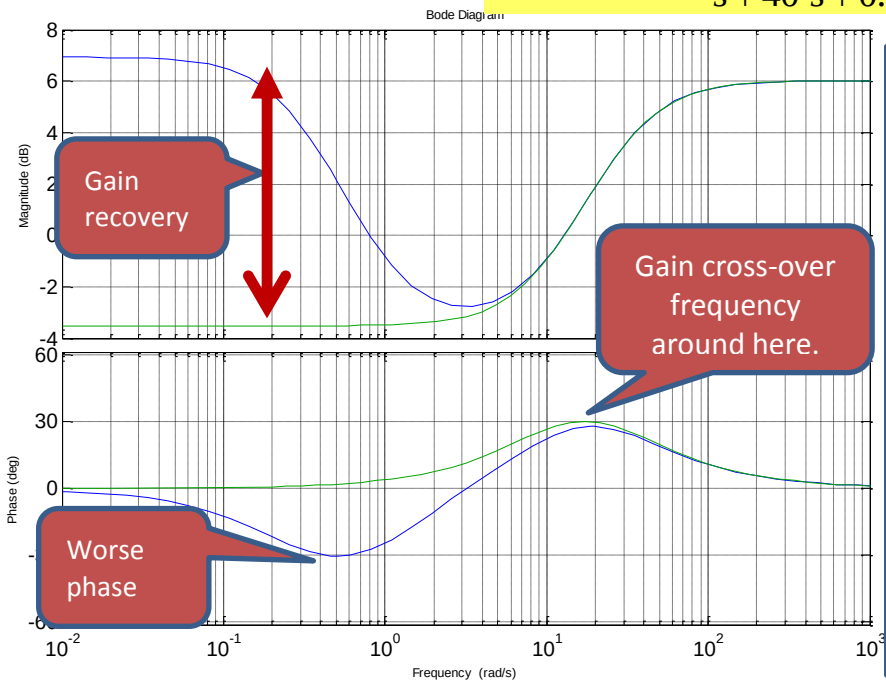
The lead has a positive phase characteristic which can be used to improve phase margins.

The negative phase from the lag component is well to the left and hence not near the gain cross over frequency.



COMPARE LEAD-LAG TO LEAD ONLY

$$Lead\_lag = 2\frac{s+10}{s+40}\frac{s+1}{s+0.3}; \quad Lead = 2\frac{s+10}{s+40}$$



Just lead has worse low frequency gain but slightly better phase.

However, phase characteristic well to left of gain cross-over frequency.

KEY ATTRIBUTES (Gain and phase)

$$K\frac{s+a}{s+\beta a}\frac{s+\gamma b}{s+b}; \quad \begin{matrix} 1 \le \beta \le 10 \\ 1 \le \gamma \le 10 \end{matrix}; \quad a > b$$

1. Steady state gain is Kγ/β, mid frequency gain is K/β and high frequency gain is K. Lead only would have a steady-state gain of K/β which is smaller.
2. Phase is zero at high and low frequency.
3. Phase is positive around corner frequencies of lead component and negative around corner frequencies of lag component.
4. Lead corner frequencies larger than lag corner frequencies.