Modelling and control summaries

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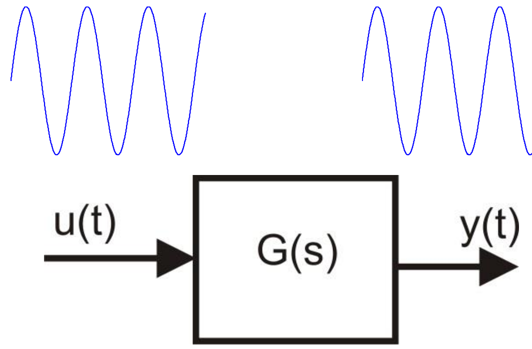
Bode 17: Quadratic factors and resonance



SUMMARY of Frequency response

$$u = D\sin(wt) \quad \Rightarrow \quad y = DA\sin(wt + \phi)$$

1. Gain A(w) is the ratio of output amplitude of oscillation to that of the input.
2. Phase φ(w) is the phase difference between the input and output responses.

Computation of A(w) and φ(w) with quadratic factors

When G(s) includes complex poles/zeros, one can not determine the gain and phase as easily.

$$A = |G(jw)|; \quad \phi = \arg(G(jw))$$

$$G(s) = \frac{4}{s^2 + s + 2}$$

NO OBVIOUS CORNER FREQUENCY AS POLES AT -0.5+1.32i, -0.5-1.32i

$$G(jw) = \frac{4}{(jw)^2 + jw + 2} = \frac{4}{-w^2 + jw + 2}$$

$$A = \sqrt{\frac{16}{(2 - w^2)^2 + w^2}}; \quad \phi = -\tan^{-1}\frac{w}{2 - w^2}$$

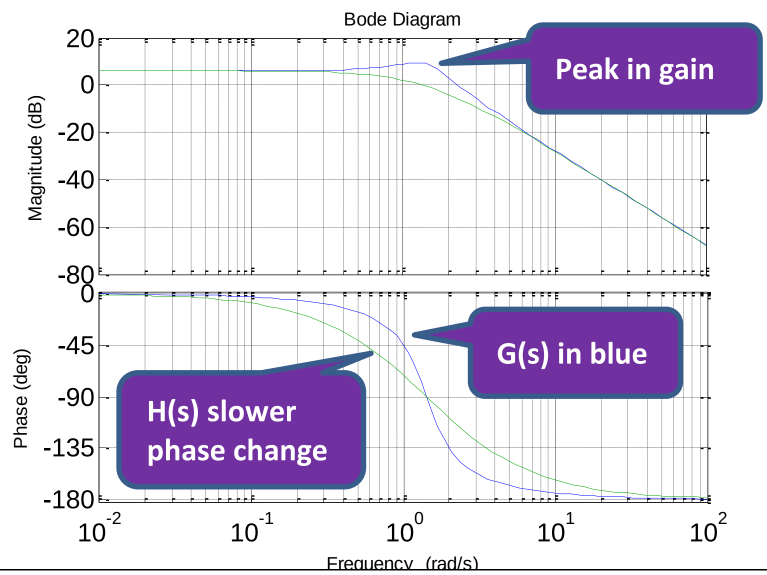
Observations

The asymptotic information (high and low frequency) is still simple (e.g. -40db/dec at high frequency)

The difficulty is that one cannot easily form a pattern for how the gain and phase change.

Here the gain plot has a peak not observed for systems with simple real poles such as in H(s). Moreover, the phase changes much more quickly for G(s) than H(s).

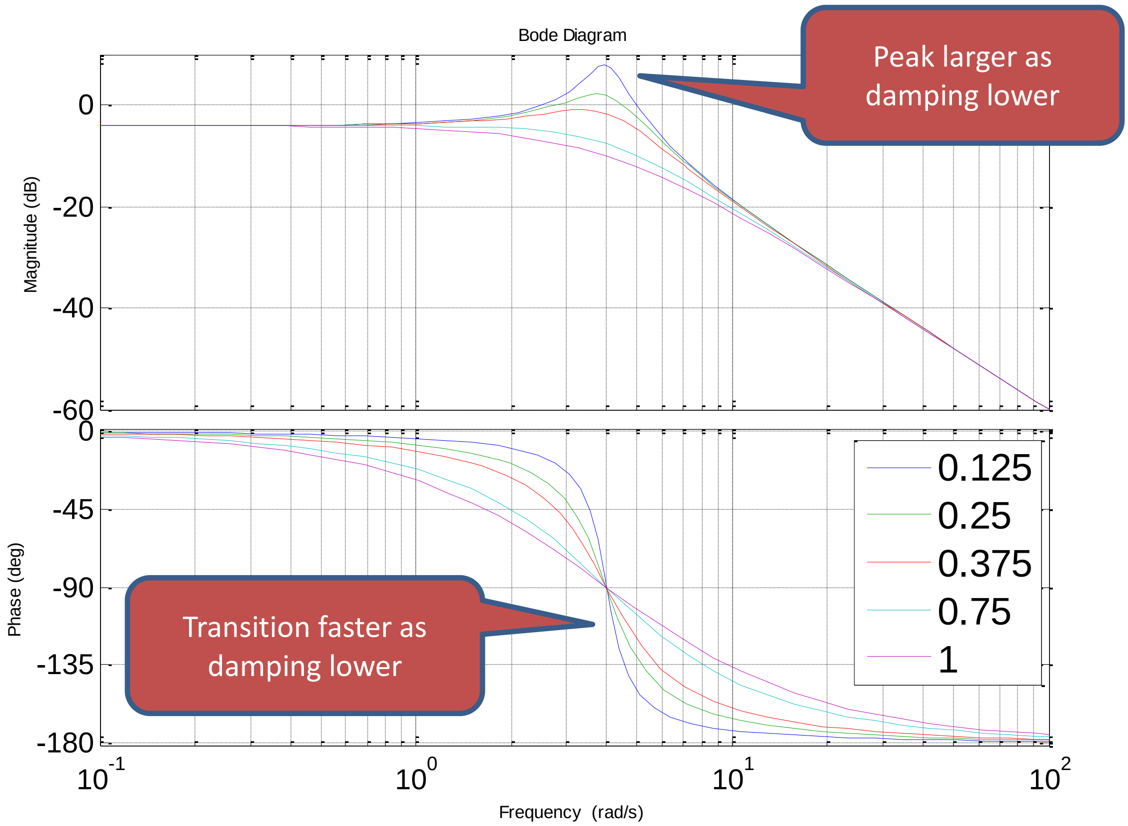
$$H(s) = \frac{4}{s^2 + 3s + 2}$$



IMPACT OF UNDERDAMPING

Consider an underdamped 2nd order system and produce bode diagrams for various damping ratios.

$$G = \frac{10}{s^2 + 2\varsigma w_n s + w_n^2}; \quad w_n = 4$$



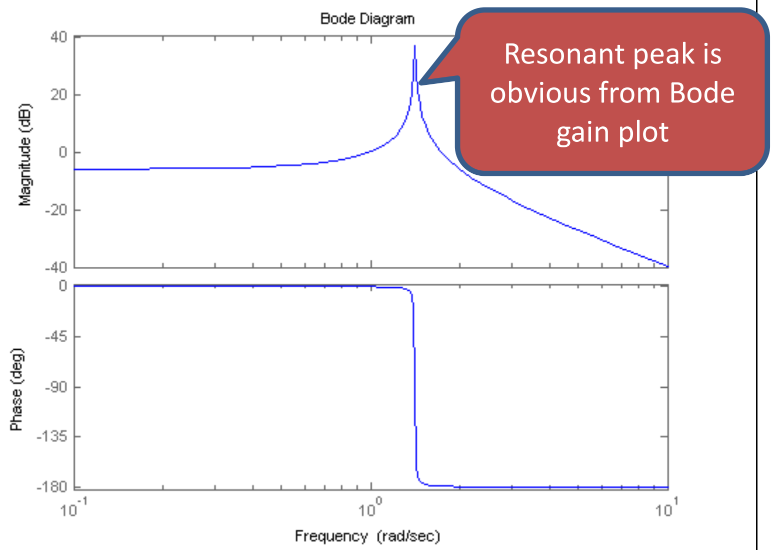
Resonant peak can be shown to occur at $w_p = w_n\sqrt{1-2\varsigma^2}$

RESONANCE

This has a loose definition.
When the gain is locally very large compared to the values in the neighbouring frequencies. This is obvious in the gain plot as seen here.

WARNING: The presence of under-damped poles need not imply a resonance exists, but it is an indicator that one such is likely to exist.

THE existence of a resonance is a problem in general as a system could shake itself to bits if excited by a frequency near this.



CONCLUSION: For a system with complex poles/zeros, it will often be easier to use software to generate the Bode plots. While one could formulate some ‘hand sketching’ guidance, it is likely to be more effort than it is worth, especially for systems with multiple poles and zeros.