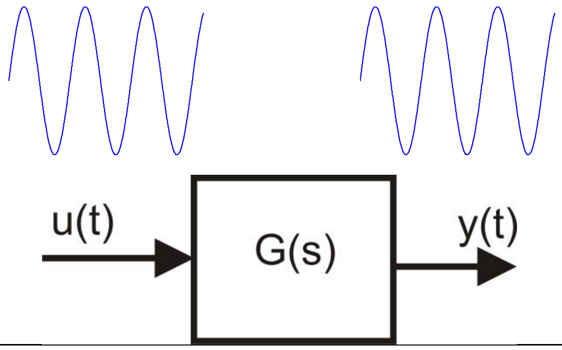
Modelling and control summaries

by Anthony Rossiter

Bode 18: bandwidth



SUMMARY of Frequency response

$$u = D\sin(wt) \quad \Rightarrow \quad y = DA\sin(wt+\phi)$$

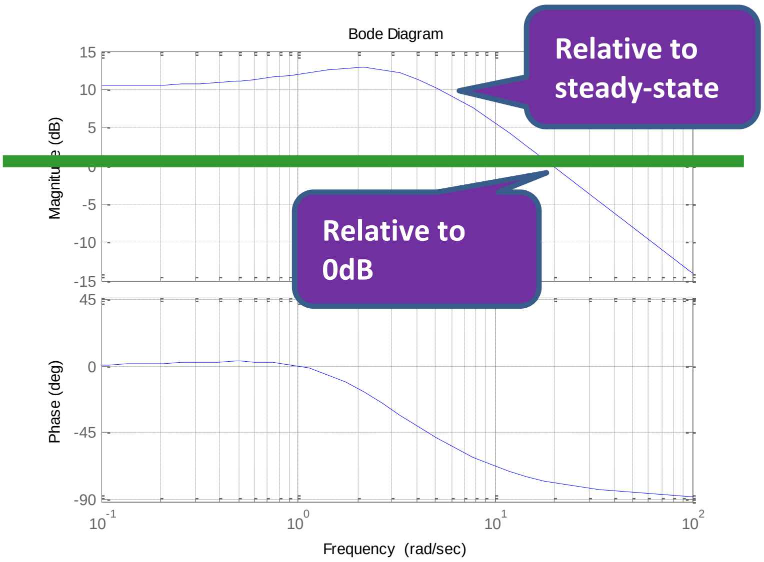
1. Gain A(w) is the ratio of output amplitude of oscillation to that of the input.
2. Phase φ(w) is the phase difference between the input and output responses.

What is bandwidth?

Typically, bandwidth is the range of frequencies for which gain A is significant.
However this is not precise enough.

Is significant relative to steady-state gain or relative to 0dB (gain of 1)? As seen here these alternatives give very different values of frequency.

A common default is that bandwidth is frequency range where gain>1/√2 (or -3dB). Use with caution!

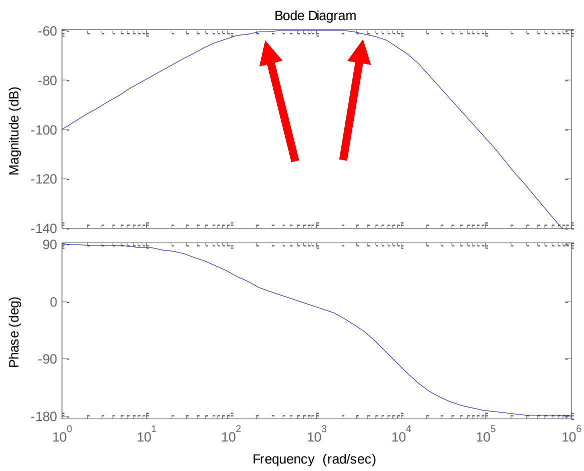


EXAMPLE of a SPEAKER

Clearly, the speaker gives a consistent reproduction for a limited range of frequencies (about 100-3000rad/s) as marked in the figure.

$$G = \frac{2.5\,s}{4e\text{-}005\,s^3 + 0.604\,s^2 + 2554\,s + 250000}$$

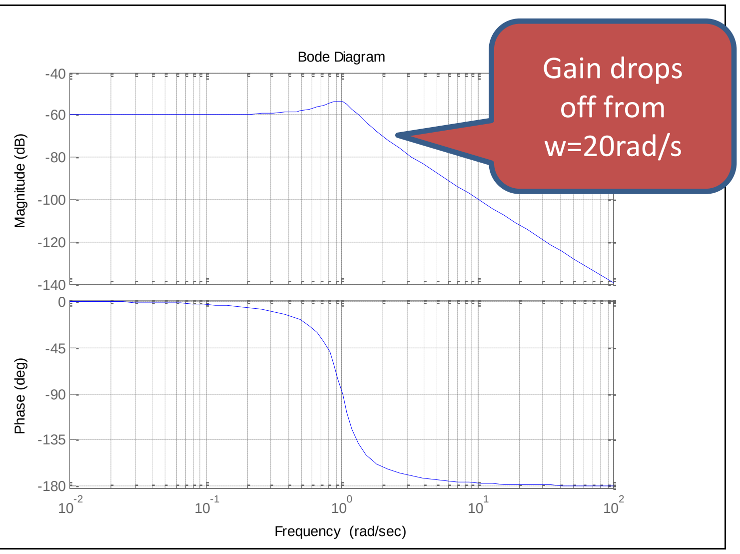
NOTE: The actual gain here is -60dB, so not close to 0dB!



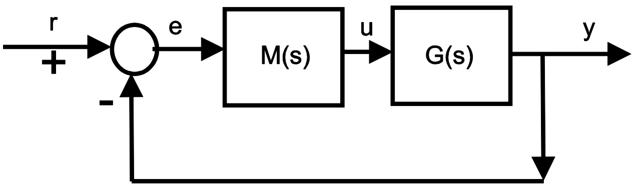
A suspension system is designed to reject high frequency disturbances (those beyond 50 rad/s). Comment on the efficacy of the unit.

$$G=\frac{s}{Ms^2+Bs+k};\quad M=10^3;\ B=500;\quad k=10^3$$

Clearly for frequencies above 20rad/s, the gain is down by more than 10dB from steady-state so fit for purpose.



Bandwidth with feedback control systems



In a typical feedback loop, the desired closed-loop gain (steady-state) is unity, that is the output tracks the target.

$$y=\frac{GM}{1+GM}r=G_c r;\quad E[G_c(0)]=1$$

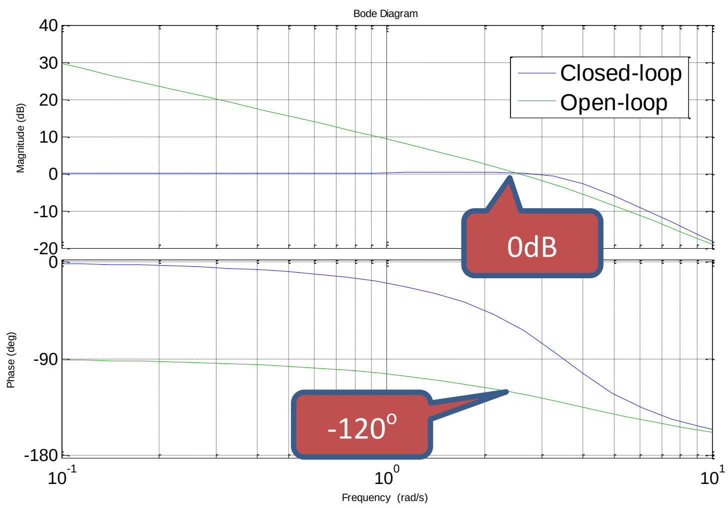
A meaningful definition of bandwidth is where the gain is greater than 1/√2 (or -3dB)

Bandwidth with open-loop or closed-loop Bode diagrams

1. **While one can use software to plot the closed-loop Bode, most design is based on the open-loop Bode and hence it is convenient to estimate closed-loop bandwidth from the open-loop Bode diagram.**
2. **Gain cross-over frequency is where the open-loop Bode is 0dB.**
3. **For a good design, the phase is about -120^o at the gain cross-over frequency.**

Substituting in the expected value of GM at the gain cross-over frequency one finds the open and closed-loop are close to 0dB at the same frequency.

$$GM\approx(-1-j\sqrt{3})/2=1\angle -120^o$$
$$\Rightarrow\quad (1+GM)=(1-j\sqrt{3})/2$$
$$\Rightarrow\quad |1+GM|=|GM|$$
$$\Rightarrow\quad \left|\frac{GM}{1+GM}\right|=1\quad(0dB)$$



A helpful approximation to closed-loop bandwidth is the gain cross over frequency.