Modelling and control summaries

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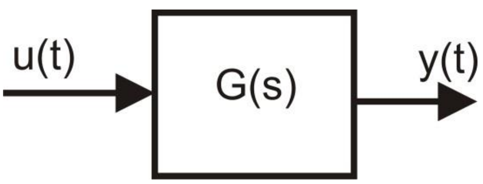
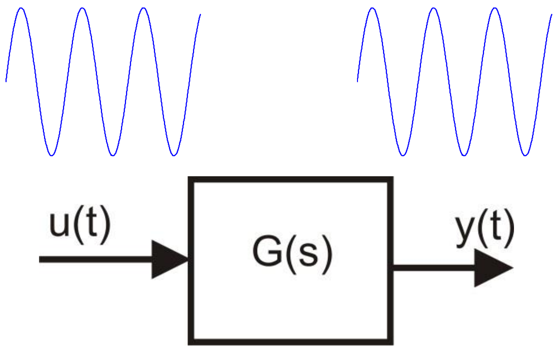
Bode 2: computing frequency response

This brief summary assumes readers are familiar with the concept of feedback, transfer functions and block diagrams. For now this series assumes the open-loop arrangement below (u(t) the input, y(t) the output).



$$U(s) = L[u(t)]$$
$$Y(s) = L[y(t)]$$
$$Y(s) = G(s)U(s)$$



SUMMARY of Frequency response

$$u = D\sin(wt) \quad \Rightarrow \quad y = DA\sin(wt + \phi)$$

1. Gain A(w) is the ratio of output amplitude of oscillation to that of the input.
2. Phase φ(w) is the phase difference between the input and output responses.

[This observation is asymptotically]

Efficient computation of A(w) and φ(w)

It can be shown that the frequency response parameters have explicit and simple links to the transfer function G(s).
That is, simply substitute s=jw and determine the complex number which results.

$$A = |G(jw)|;$$
$$\phi = \arg(G(jw))$$

Find the frequency response for the following system:

$$G(s) = \frac{4}{s^2 + 3s + 2}$$

$$G(jw) = \frac{4}{3jw + 2 - w^2}$$

$$A = \sqrt{\frac{16}{(2-w^2)^2 + 9w^2}}; \quad \phi = -\tan^{-1}\frac{3w}{2-w^2}$$

At w=1

$$A = \sqrt{\frac{16}{1+9}}; \quad \phi = -\tan^{-1}\frac{3}{1}$$

At w=2

$$A = \sqrt{\frac{16}{40}}; \quad \phi = -\pi + \tan^{-1}\frac{3}{2}$$

Find the frequency response for the following system:

$$G(s)=\frac{2s+1}{s^3+s^2+3s+2}$$

WARNING: **When calculating the phase, think carefully about which quadrant the complex number lies in.**

$$G(jw)=\frac{2jw+1}{(jw)^3+(jw)^2+3jw+2}=\frac{2jw+1}{j(3w-w^3)+2-w^2}$$

$$|G(jw)|=\frac{\sqrt{4w^2+1}}{\sqrt{(3w-w^3)^2+(2-w^2)^2}}$$

$$\angle G(jw)=\tan^{-1}\frac{2w}{1}-\tan^{-1}\frac{(3w-w^3)}{2-w^2}$$

QUESTIONS: Find the frequency response parameters for the following systems

$$G(s)=\frac{3}{s+2};\quad G(s)=\frac{s+4}{s^2+6s+2};\quad G(s)=\frac{s^2+s+2}{s^4+s^3+4s^2+2};\quad G(s)=\frac{s-1}{s(s+4)}$$

USING MATLAB TO CHECK ANSWERS

1. The main MATLAB command is bode.m.
2. This will work with objects defined as transfer functions using tf.m
3. An example is given here. The phase is given in degrees and not radians.

The command supplies the transfer function G and the required frequency. This example uses w=3.