Modelling and control summaries

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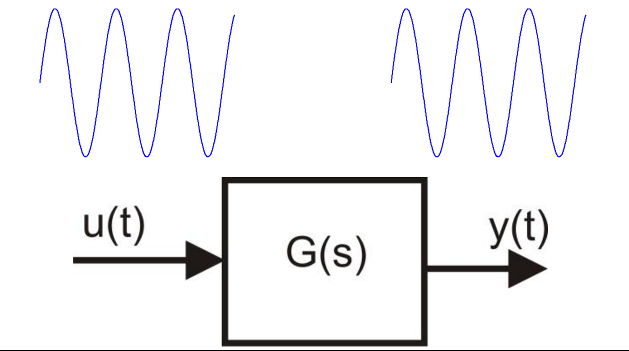
Bode 3: frequency response with factors



SUMMARY of Frequency response

$$u = D\sin(wt) \quad \Rightarrow \quad y = DA\sin(wt+\phi)$$

1. Gain A(w) is the ratio of output amplitude of oscillation to that of the input.
2. Phase φ(w) is the phase difference between the input and output responses.

Multiplication and division of complex numbers

1. Modulus of the product is the product of the moduli.
2. Phase (or argument) of the product is the sum of the phases.

$$|wzy| = |w||z||y|; \quad \left|\frac{wz}{ykm}\right| = \frac{|w||z|}{|y||k||m|}; \quad \left|\frac{w^2 z}{y^3 k}\right| = \frac{|w|^2|z|}{|y|^3|k|}$$

$$\angle wzy = \angle w + \angle z + \angle y; \quad \angle\frac{w^2 z}{y^3 k} = 2\angle w + \angle z - 3\angle y - \angle k$$

Frequency response for a single factor of the form (s+a) or 1/(s+b)

Gain and phase of single factors can be determined by inspection.

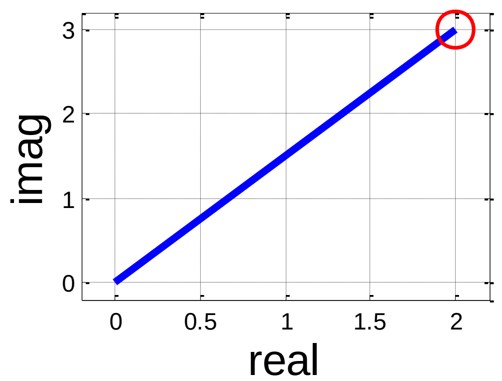
$$\angle(jw+a) = \tan^{-1}\frac{w}{a}; \quad |jw+a| = \sqrt{w^2+a^2}$$

$$\angle\frac{1}{jw+b} = -\tan^{-1}\frac{w}{b}; \quad \left|\frac{1}{jw+b}\right| = \frac{1}{\sqrt{w^2+b^2}}$$

Plot on an Argand diagram if this helps.

Here we plot 2+3j. It is clear that the gain and phase are:

$$gain = \sqrt{2^2+3^2}; \qquad \phi = \tan^{-1}\frac{3}{2}$$



Frequency response with two factors uses the simple properties of complex numbers $G(s)=\frac{s+a}{s+b}$	$\phi=\angle\frac{jw+a}{jw+b}=\tan^{-1}\frac{w}{a}-\tan^{-1}\frac{w}{b};$ $A=\left\|\frac{w+a}{jw+b}\right\|=\frac{\sqrt{w^2+a^2}}{\sqrt{w^2+b^2}}$

Frequency response with three factors uses the simple properties of complex numbers $G(s)=\frac{s+2}{(s+3)(s+4)}$	$\phi=\angle\frac{jw+2}{(jw+3)(jw+4)}=\tan^{-1}\frac{w}{2}-\tan^{-1}\frac{w}{3}-\tan^{-1}\frac{w}{4}$ $A=\left\|\frac{jw+2}{(jw+3)(jw+4)}\right\|=\frac{\sqrt{w^2+4}}{\sqrt{w^2+9}\sqrt{w^2+16}}$

Frequency response with an integrator $G(s)=\frac{4}{s(s+4)}$ **WARNING**: An integrator has a fixed phase as **s=jw** is always purely imaginary.	$\phi=\angle\frac{4}{(jw)(jw+4)}=-\frac{\pi}{2}-\tan^{-1}\frac{w}{4}$ $A=\left\|\frac{4}{(jw)(jw+4)}\right\|=\frac{4}{\sqrt{w^2}\sqrt{w^2+16}}=\frac{4}{w\sqrt{w^2+16}}$

QUESTIONS: By first factorising, find the gain and phase of the following systems

$$G(s)=\frac{3}{s+2};\quad G(s)=\frac{s+4}{s^2+4s+3};$$

$$G(s)=\frac{s^2+3s+2}{s^3+9s^2+20s};\quad G(s)=\frac{3(s+0.1)}{s(s+0.4)(s+0.3)}$$

USING MATLAB TO CHECK ANSWERS

1. The main MATLAB command is bode.m.
2. This will work with objects defined as transfer functions using tf.m
3. An example is given here. The phase is given in degrees and not radians.

The command supplies the transfer function G and the required frequency. This example uses w=3.