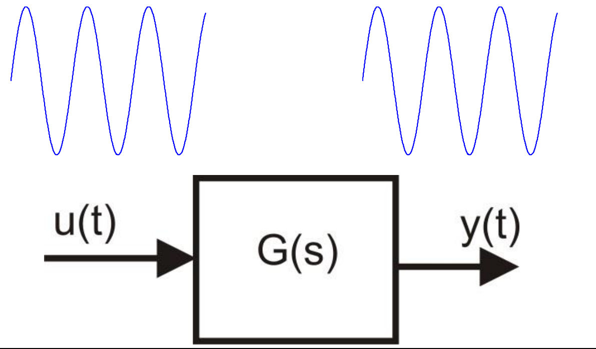
Modelling and control summaries

by Anthony Rossiter

The University Of Sheffield.

Bode 4: frequency response & RHP factors



SUMMARY of Frequency response

$$u = D\sin(wt) \quad \Rightarrow \quad y = DA\sin(wt+\phi)$$

1. Gain A(w) is the ratio of output amplitude of oscillation to that of the input.
2. Phase φ(w) is the phase difference between the input and output responses.

Frequency response for a single LHP factor of the form (s+a) or 1/(s+b)

Gain and phase of single LHP factors can be determined by inspection.

$$\angle(jw+a) = \tan^{-1}\frac{w}{a}; \quad |jw+a| = \sqrt{w^2+a^2}$$

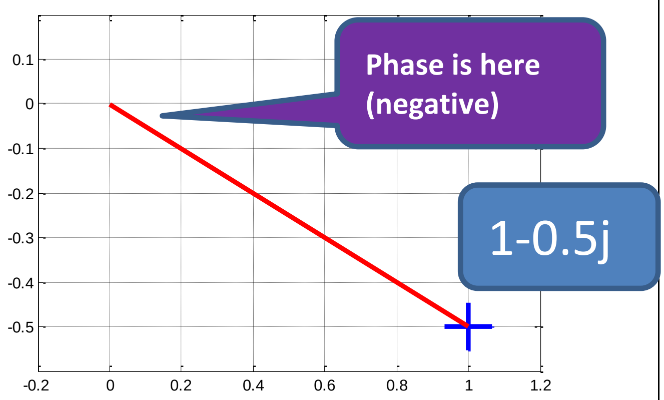
$$\angle\frac{1}{jw+b} = -\tan^{-1}\frac{w}{b}; \quad \left|\frac{1}{jw+b}\right| = \frac{1}{\sqrt{w^2+b^2}}$$

Frequency response for a single RHP factor of the form (s-a) or 1/(s-b)

A lot more care is needed with RHP factors as inverse tan has two possible solutions within 0 to 360 degrees.

RECOMMENDATION: Always plot the underlying complex number in an Argand diagram to ensure you compute phase in the correct quadrant. [Use inverse tan so always returns 0 to 90 degrees]

$$\angle(1-0.5j) = -\tan^{-1}\frac{0.5}{1}$$



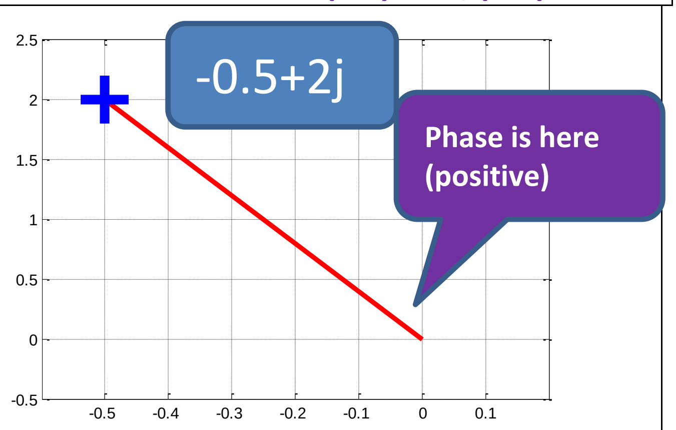
REMARK: If there is a RHP factor in a denominator, the easiest thing to do is find the phase of the factor and then apply a minus sign.

$$\angle\frac{1}{z} = -\angle z; \qquad \angle\left(\frac{1}{1-0.5j}\right) = -\angle(1-0.5j) = \tan^{-1}0.5$$

Frequency response for a single RHP factor of the form (s-a) or 1/(s-b)

RECOMMENDATION: Always plot the underlying complex number in an Argand diagram to ensure you compute phase in the correct quadrant. [Use inverse tan so always returns 0 to 90 degrees]

$$\angle(-0.5+2j) = 180 - \tan^{-1}\frac{2}{0.5}$$



SUMMARY	$\angle(a-jw) = -\tan^{-1}\frac{w}{a}$ $\quad \angle(-a+jw) = 180 - \tan^{-1}\frac{w}{a}$ $\angle\frac{1}{a-jw} = \tan^{-1}\frac{w}{a}$ $\quad \angle\frac{1}{-a+jw} = -180 + \tan^{-1}\frac{w}{a}$

Find the gain and phase $G = \frac{5}{s-2\sqrt{3}}$	$\phi = \angle\frac{5}{(jw-2\sqrt{3})} = -180 + \tan^{-1}\frac{w}{2\sqrt{3}}$ $A = \left\vert\frac{5}{(jw-2\sqrt{3})}\right\vert = \frac{5}{\sqrt{w^2+4\times 3}}$

Find the gain and phase $G = \frac{6(s+2)}{(3-s)(s+4)}$	$\phi = \angle\frac{jw+2}{(3-jw)(jw+4)} = \tan^{-1}\frac{w}{2} - \tan^{-1}\frac{w}{4} + \tan^{-1}\frac{w}{3}$ $A = \left\vert\frac{5(jw+2)}{(3-jw)(jw+4)}\right\vert = \frac{5\sqrt{w^2+4}}{\sqrt{w^2+9}\sqrt{w^2+16}}$

<u>QUESTIONS: By first factorising, find the gain and phase of the following systems; Use MATLAB to check your answers</u>

$$G(s) = \frac{3}{2-s}; \quad G(s) = \frac{s-4}{s^2+2s-3};$$

$$G(s) = \frac{s^2-3s+2}{s^3+9s^2+20s}; \quad G(s) = \frac{3(s-0.1)}{s(0.4-s)(s+0.3)}$$