Modelling and control summaries

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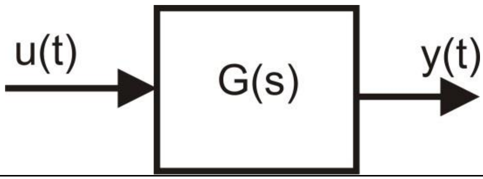
Bode 7: Frequency response on log scales



SUMMARY of Frequency response

We have established that frequency response is given by the formulae:

$$gain = |G(jw)|; \quad phase = \angle G(jw)$$

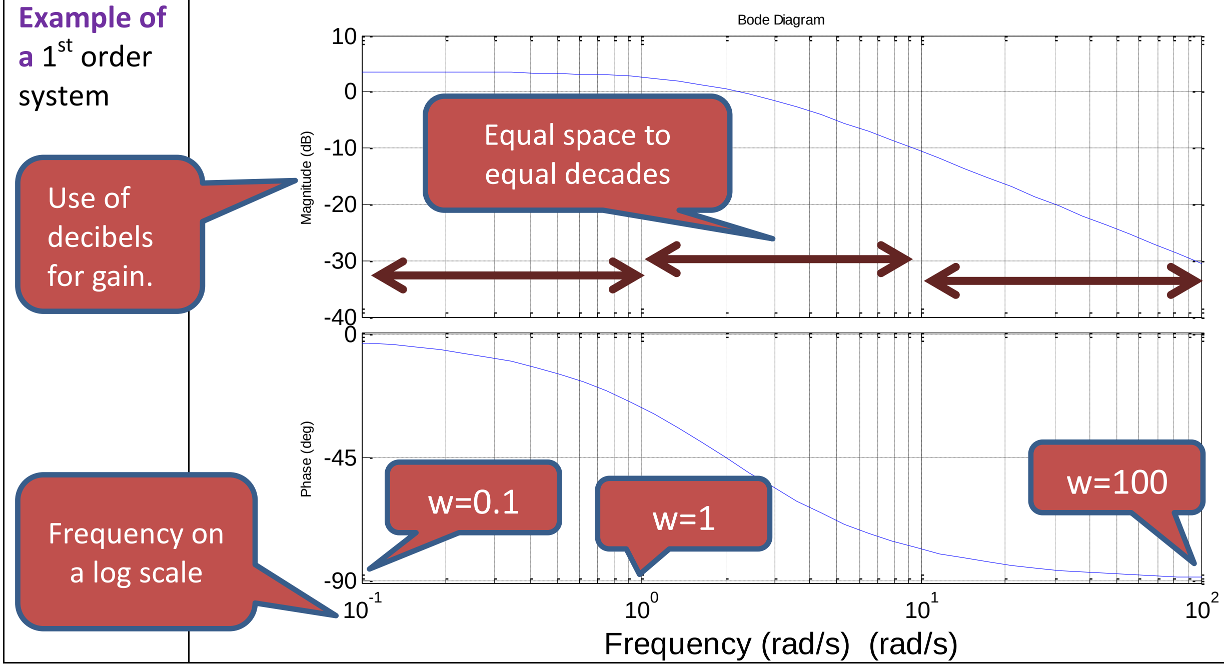
Phase normal expressed in degrees.

A plot of w against gain or phase, while helpful, does not display good enough detail at low or high frequency, thus a logarithmic scale is used for frequency to ensure that each decade (0.1-1, 1-10,10-100,etc.) receives the same space on the graph.

BODE DIAGRAM (actually this comprises two plots)

1. **A plot of $\log_{10}(w)$ vs $20\log_{10}|G(jw)|$ (denoted decibels)**
2. **A plot of $\log_{10}(w)$ vs arg(G(jw)) (usually in degrees)**

Example of a 1st order system



REMARKS

1. A decade is a factor of 10 change in frequency. Each decade in frequency has the same space on the diagram (see double sided arrows for decades 0.01 to 0.1, 0.1 to 1, 1 to 10, 10 to 100).
2. Gain and phase PLOTS are best aligned on top of each other as this helps with design.

UNDERSTANDING LOGARITHMS	[All below log to base 10]
To work with Bode diagrams, a student must be fluent in the algebra of logarithms and in key formulae shown here.	$\log(ab) = \log(a) + \log(b)$ $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$
A change of 20dB is equivalent to a change in gain of a factor of 10. • 20dB up is multiply by 10. • 20dB down is divide by 10.	$20\log(a) = \gamma \quad \Rightarrow \quad 20\log(10a) = \gamma + 20\log(10) = \gamma + 20$ $20\log(a) = \gamma \quad \Rightarrow \quad 20\log(\frac{a}{10}) = \gamma - 20\log(10) = \gamma - 20$
A change in gain of 3dB is equivalent to a change in gain of a factor of √2 (approx).	$20\log(a) = \gamma \quad \Rightarrow \quad 20\log(a\sqrt{2}) = \gamma + 20\log(\sqrt{2}) \approx \gamma + 3$ $20\log(a) = \gamma \quad \Rightarrow \quad 20\log(\frac{a}{\sqrt{2}}) = \gamma - 20\log(\sqrt{2}) \approx \gamma - 3$
It is also useful, for insight and estimation (back of envelope computations) to know logarithms of key integers and factors of 10.	$\log(1) = 0, \quad \log(2) \approx 0.3\,(6dB), \quad \log(3) \approx 0.48\,(10dB),$ $\log(5) \approx 0.7\,(14dB), \quad \log(6) \approx 0.8\,(16dB), \quad \log(8) \approx 0.9\,(18dB),$ $\log(10) = 1\,(20dB), \quad \log(100) = 2\,(40dB), \quad \log(0.01) = -2\,(-40dB),$

Bode diagram summary

1. If you need a precise plot – USE A COMPUTER!!

2. These notes are focussed on sketching because an understanding of how to sketch is important for an understanding of control design.

3. A key point about sketching is that it is meant to be quick and simple – if you are doing lots of number crunching then USE A COMPUTER!

4. When sketching, it is permissible to use reasonable approximation, that is values to within 5% are often more than sufficient. Do not waste time with several decimal places/significant figures.