A plot of $w$ against gain or phase, while helpful, does not display good enough detail at low or high frequency, thus a logarithmic scale is used for frequency to ensure that each decade (0.1-1, 1-10, 10-100, etc.) receives the same space on the graph.

**BODE DIAGRAM (actually this comprises two plots)**

1. A plot of $\log_{10}(w)$ vs $20\log_{10}|G(jw)|$ (denoted decibels)
2. A plot of $\log_{10}(w)$ vs $\text{arg}(G(jw))$ (usually in degrees)

**REMARKS**

1. A decade is a factor of 10 change in frequency. Each decade in frequency has the same space on the diagram (see double sided arrows for decades 0.01 to 0.1, 0.1 to 1, 1 to 10, 10 to 100).
2. Gain and phase PLOTS are best aligned on top of each other as this helps with design.
**UNDERSTANDING LOGARITHMS**

<table>
<thead>
<tr>
<th><strong>To work with Bode diagrams, a student must be fluent in the algebra of logarithms and in key formulae shown here.</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \log(ab) = \log(a) + \log(b) ]</td>
</tr>
<tr>
<td>[ \log\left(\frac{a}{b}\right) = \log(a) - \log(b) ]</td>
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</tbody>
</table>

A change of 20dB is equivalent to a change in gain of a factor of 10.

- 20dB up is multiply by 10.
- 20dB down is divide by 10.

\[
\begin{align*}
20\log(a) = \gamma & \implies 20\log(10a) = \gamma + 20\log(10) = \gamma + 20 \\
20\log(a) = \gamma & \implies 20\log\left(\frac{a}{10}\right) = \gamma - 20\log(10) = \gamma - 20
\end{align*}
\]

A change in gain of 3dB is equivalent to a change in gain of a factor of \(\sqrt{2}\) (approx).

\[
\begin{align*}
20\log(a) = \gamma & \implies 20\log(a\sqrt{2}) = \gamma + 20\log(\sqrt{2}) \approx \gamma + 3 \\
20\log(a) = \gamma & \implies 20\log\left(\frac{a}{\sqrt{2}}\right) = \gamma - 20\log(\sqrt{2}) \approx \gamma - 3
\end{align*}
\]

It is also useful, for insight and estimation (back of envelope computations) to know logarithms of key integers and factors of 10.

\[
\begin{align*}
\log(1) &= 0, & \log(2) &\approx 0.3\ (6dB), & \log(3) &\approx 0.48\ (10dB), \\
\log(5) &= 0.7\ (14dB), & \log(6) &\approx 0.8\ (16dB), & \log(8) &\approx 0.9\ (18dB), \\
\log(10) &= 1\ (20dB), & \log(100) &= 2\ (40dB), & \log(0.01) &=-2\ (-40dB),
\end{align*}
\]

---

**Bode diagram summary**

1. If you need a precise plot – **USE A COMPUTER!!**

2. These notes are focussed on sketching because an understanding of how to sketch is important for an understanding of control design.

3. A key point about sketching is that it is meant to be quick and simple – if you are doing lots of number crunching then **USE A COMPUTER!**

4. When sketching, it is permissible to use reasonable approximation, that is values to within 5% are often more than sufficient. Do not waste time with several decimal places/significant figures.