SUMMARY of Frequency response
We have established that frequency response is given by the formulae:
\[ \text{gain} = |G(jw)|; \quad \text{phase} = \angle G(jw) \]
Phase normal expressed in degrees.

A plot of \( w \) against gain or phase, while helpful, does not display good enough detail at low or high frequency, thus a logarithmic scale is used for frequency to ensure that each decade (0.1-1, 1-10,10-100,etc.) receives the same space on the graph.

BODE DIAGRAM (actually this comprises two plots)
1. A plot of \( \log_{10}(w) \) vs \( 20\log_{10}|G(jw)| \) (denoted decibels)
2. A plot of \( \log_{10}(w) \) vs \( \arg(G(jw)) \) (usually in degrees)

Example 1
A simple differentiator.
\[
G(s) = s; \quad |G(jw)| = w; \quad \angle G(jw) = 90^\circ
\]

REMARKS
1. Gain plot has a slope of 20dB/decade; a decade as a factor of 10 change in frequency.
2. Each decade in frequency has the same space on the diagram (see double sided arrows for decades 0.01 to 0.1, 0.1 to 1, 1 to 10, 10 to 100).
HERON plots are best done using asymptotic behaviour (high and low frequency) as illustrated in following examples. A correction is used at a mid-frequency to join asymptotes.
Example 2
A simple integrator.
\[ G(s) = \frac{1}{s} \]
\[ |G(jw)| = \frac{1}{w} \]
\[ \angle G(jw) = -90^\circ \]
GAIN slope is clearly -20dB/dec.

Example 3
A simple zero.
\[ G(s) = s + 2; \]
\[ |G(jw)| = \sqrt{w^2 + 4}; \]
\[ \angle G(jw) = \tan^{-1} \frac{w}{2} \]
\[ \begin{aligned} w = 2, & |G(jw)| = 2, \\ \angle G(jw) &= 45^\circ \end{aligned} \]
Asymptotic GAIN slope is 20dB/dec.

Example 4
A simple pole.
\[ G(s) = \frac{1}{s+4}; \]
\[ |G(jw)| = \frac{1}{\sqrt{w^2 + 16}} \]
\[ \angle G(jw) = -\tan^{-1} \frac{w}{4} \]
\[ \begin{aligned} w = 4, & |G(jw)| = \frac{1}{4\sqrt{2}}, \\ \angle G(jw) &= -45 \end{aligned} \]
Asymptotic GAIN slope is clearly -20dB/dec.