Modelling and control summaries

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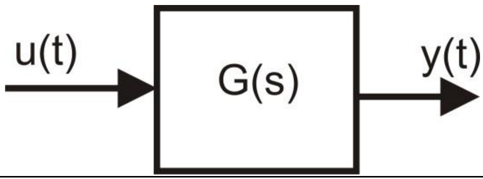
Bode 8: Bode diagrams with simple factors



SUMMARY of Frequency response

We have established that frequency response is given by the formulae:

$gain = |G(jw)|; \quad phase = \angle G(jw)$

Phase normal expressed in degrees.

A plot of w against gain or phase, while helpful, does not display good enough detail at low or high frequency, thus a logarithmic scale is used for frequency to ensure that each decade (0.1-1, 1-10,10-100,etc.) receives the same space on the graph.

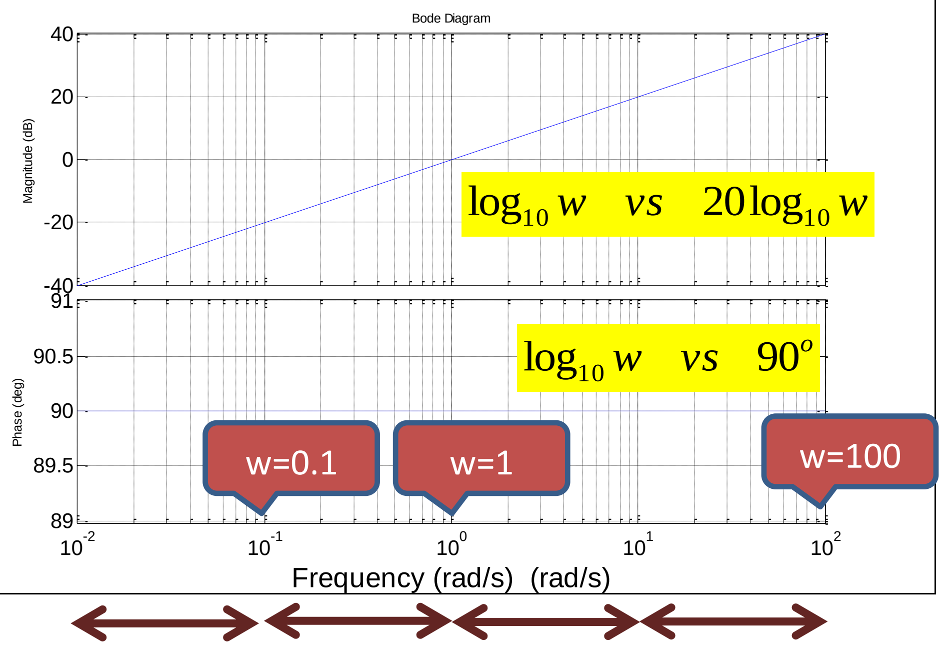
BODE DIAGRAM (actually this comprises two plots)

1. **A plot of $\log_{10}(w)$ vs $20\log_{10}|G(jw)|$ (denoted decibels)**
2. **A plot of $\log_{10}(w)$ vs $\arg(G(jw))$ (usually in degrees)**

Example 1

A simple differentiator.

$$G(s) = s;$$
$$|G(jw)| = w;$$
$$\angle G(jw) = 90^0$$



REMARKS

1. Gain plot has a slope of 20dB/decade; a decade as a factor of 10 change in frequency.
2. Each decade in frequency has the same space on the diagram (see double sided arrows for decades 0.01 to 0.1, 0.1 to 1, 1 to 10, 10 to 100).

HEREON plots are best done using asymptotic behaviour (high and low frequency) as illustrated in following examples. A correction is used at a mid-frequency to join asymptotes.

Example 2

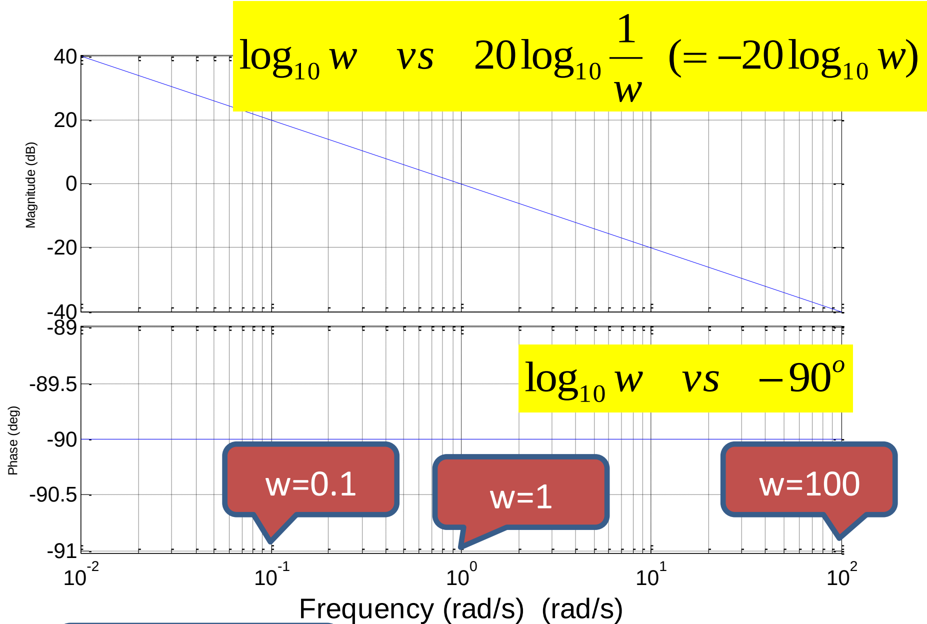
A simple integrator.

$$G(s)=\frac{1}{s};$$

$$|G(jw)|=\frac{1}{w};$$

$$\angle G(jw)=-90^0$$

GAIN slope is clearly -20dB/dec.



Example 3

A simple zero.

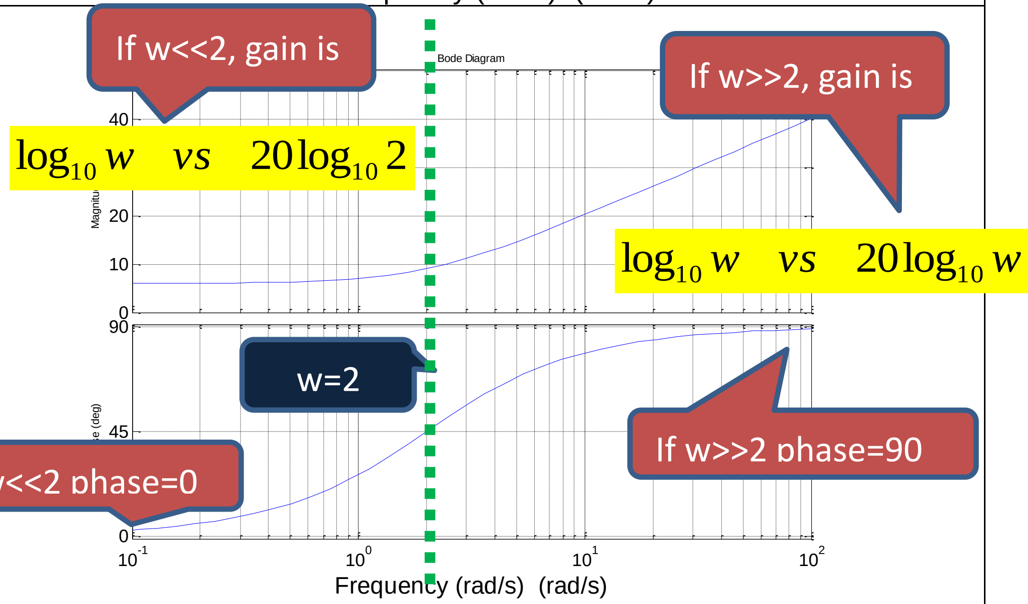
$$G(s)=s+2;$$

$$|G(jw)|=\sqrt{w^2+4};$$

$$\angle G(jw)=\tan^{-1}\frac{w}{2}$$

$$\begin{Bmatrix} w=2, |G(jw)|=2, \\ \angle G(jw)=45^o \end{Bmatrix}$$

Asymptotic GAIN slope is 20dB/dec.



Example 4

A simple pole.

$$G(s)=\frac{1}{s+4};$$

$$|G(jw)|=\frac{1}{\sqrt{w^2+16}}$$

$$\angle G(jw)=-\tan^{-1}\frac{w}{4}$$

$$\begin{Bmatrix} w=4, |G(jw)|=\frac{1}{4\sqrt{2}} \\ \angle G(jw)=-45 \end{Bmatrix}$$

Asymptotic GAIN slope is clearly -20dB/dec.