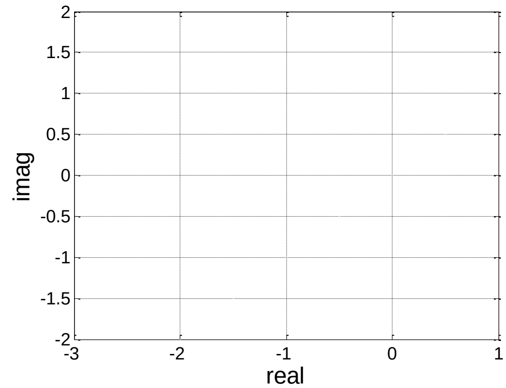
Use the first 3 rules to construct root-loci for the following

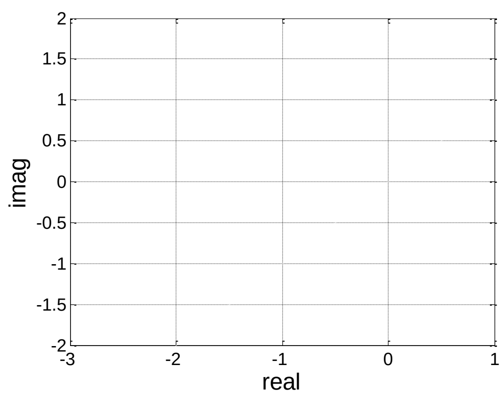
$$\left\{ \begin{array}{c} G = \dfrac{1}{s^3 + 3s^2 + 3s + 1} \\ K(s) = \dfrac{0.2(s+3)}{(s+4)} K \end{array} \right\}$$

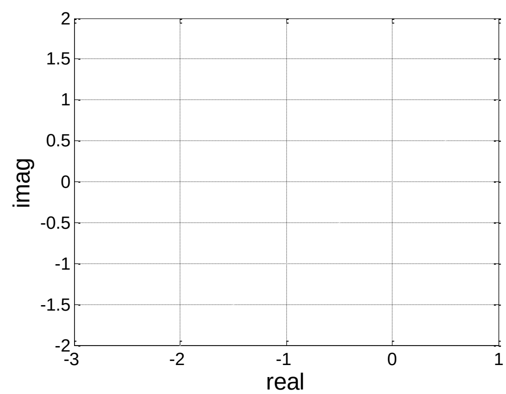
Are there any changes for positive feedback?

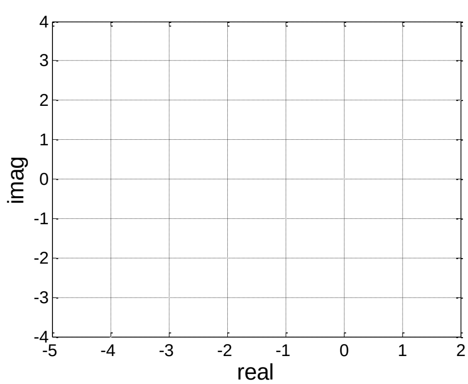
$$\left\{ \begin{array}{c} G = \dfrac{1}{s^3 + 6s^2 + 11s + 6} \\ K(s) = \dfrac{0.2}{s} K \end{array} \right\} \quad \left\{ \begin{array}{c} G = \dfrac{1}{s^2 + 3s + 2} \\ K(s) = \dfrac{K}{s} \end{array} \right\} \quad \left\{ \begin{array}{c} G = \dfrac{1}{s^3 + 6s^2 + 11s + 6} \\ K(s) = \dfrac{K}{s} \end{array} \right\}$$

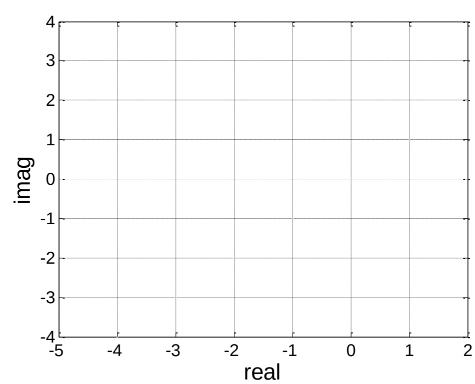
$$\left\{ \begin{array}{c} G = \dfrac{1}{s^2 + 3s + 2} \\ K(s) = K \end{array} \right\} \quad \left\{ \begin{array}{c} G = \dfrac{1}{s^3 + 6s^2 + 11s + 6} \\ K(s) = \dfrac{(s+6)}{(s+4)} K \end{array} \right\}$$











Rule 4: find the centroid for the following examples.
Are there and changes for positive feedback?

$$\left\{ \begin{array}{l} G = \dfrac{1}{s^2+3s+2} \\ K(s) = \dfrac{0.4}{s}K \end{array} \right\}$$

$$\left\{ \begin{array}{l} G = \dfrac{1}{s^3+3s^2+3s+1} \\ K(s) = \dfrac{0.2(s+3)}{(s+4)}K \end{array} \right\}$$

$$\left\{ \begin{array}{l} G = \dfrac{1}{s^3+6s^2+11s+6} \\ K(s) = \dfrac{0.2}{s}K \end{array} \right\}$$

Rule 5: Where are loci on the real axis?

$$\left\{ \begin{array}{l} G = \dfrac{1}{s^3+3s^2+3s+1} \\ K(s) = \dfrac{0.2(s+3)}{(s+4)}K \end{array} \right\}$$

$$\left\{ \begin{array}{l} G = \dfrac{1}{s^3+6s^2+11s+6} \\ K(s) = \dfrac{0.2}{s}K \end{array} \right\}$$

Are there any changes for positive feedback?

Tutorial questions

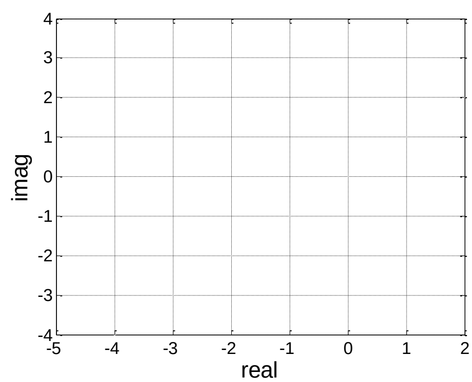
Sketch the root-loci for the following systems using rules 1-5.

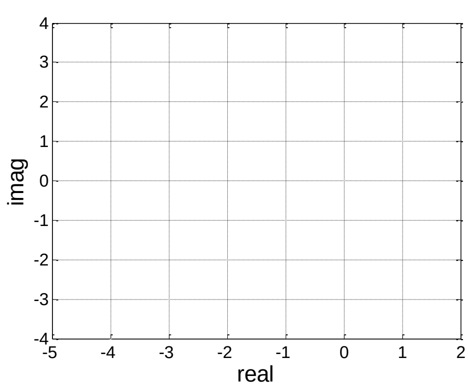
$$\left\{ \begin{array}{l} G = \dfrac{1}{s^2+2s+3} \\ K \end{array} \right\} ; \left\{ \begin{array}{l} G = \dfrac{1}{s^3+9s^2+23s+15} \\ K \end{array} \right\} ; \left\{ \begin{array}{l} G = \dfrac{(s+3)}{(s+1)(s+2)(s+4)} \\ K \end{array} \right\}$$

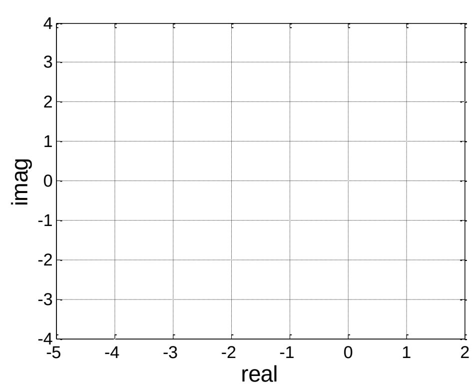
$$\left\{ \begin{array}{l} G = \dfrac{s+3}{s^2+2s+1} \\ K(s) = \dfrac{K}{s} \end{array} \right\} ; \left\{ \begin{array}{l} G = \dfrac{(s+0.5)}{s^3+9s^2+23s+15} \\ K(s) = K\dfrac{(s+4)}{s} \end{array} \right\} ; \left\{ \begin{array}{l} G = \dfrac{(s+2)}{(s+10)(s^3+4s^2+3s)} \\ K \end{array} \right\}$$

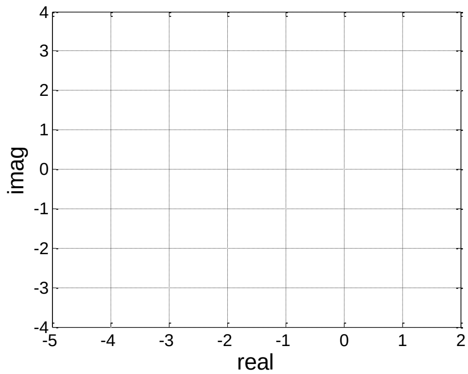
$$s^3+9s^2+23s+15 = (s+1)(s+3)(s+5)$$

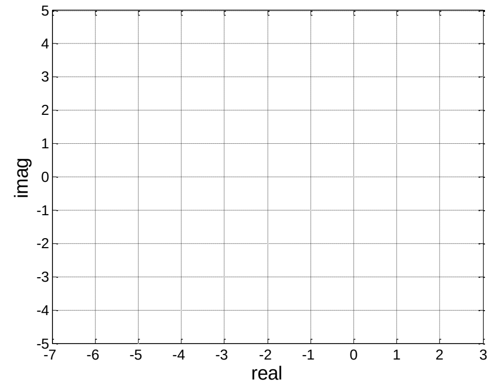
Make up examples of your own or from text books and validate your sketches with MATLAB.

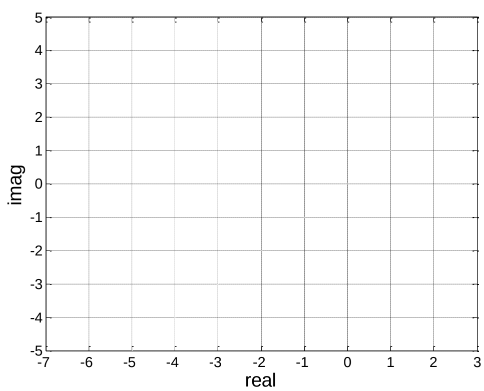


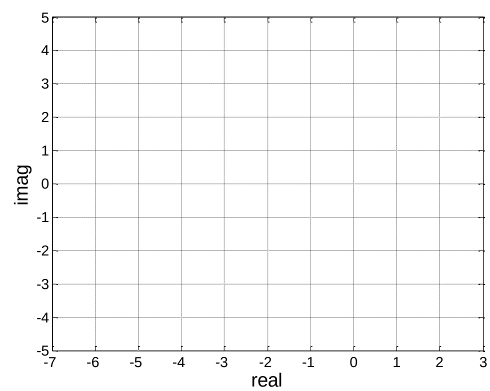












Tutorial questions

Sketch the root-loci for the following systems using negative feedback.

$$\left\{ \begin{matrix} G = \dfrac{s+4}{s^2+2s-3} \\ K \end{matrix} \right\} \quad ; \left\{ \begin{matrix} G = \dfrac{(1-s)}{s^3+9s^2+23s+15} \\ K \end{matrix} \right\} \quad ; \left\{ \begin{matrix} G = \dfrac{(s+5)}{(s+1)(s-2)(s+4)} \\ K \end{matrix} \right\}$$

$$\left\{ \begin{matrix} G = \dfrac{s+3}{-s^2+2s+3} \\ K(s) = \dfrac{K}{s} \end{matrix} \right\}; \quad s^3+9s^2+23s+15 = (s+1)(s+3)(s+5)$$

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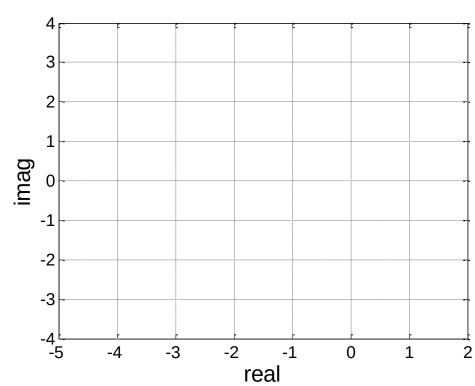

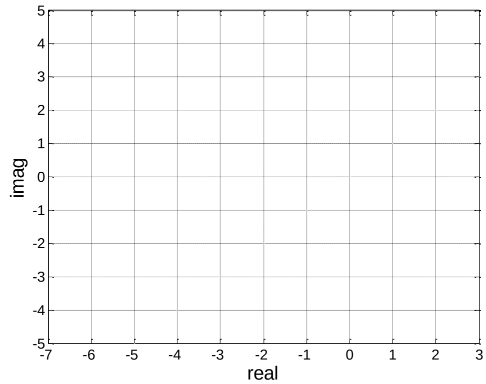

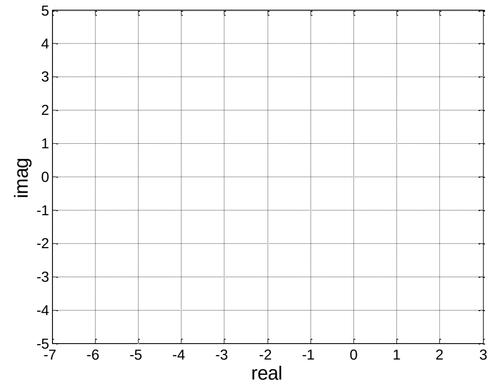

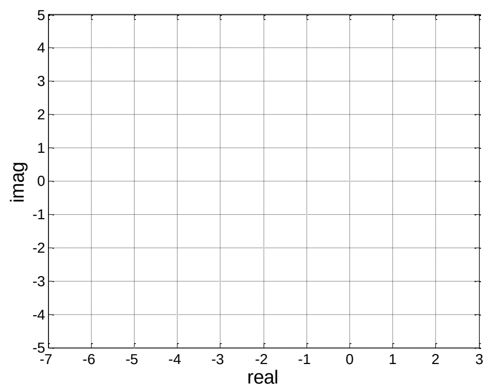

Old exam question

A control system is given by.

$$GK = \frac{K}{s(s+4)(s^2+8s+32)}; \quad K > 0$$

Sketch the root-loci showing all your working.

Determine the value of K such that the root-loci cuts the imaginary axis. What are the corresponding closed-loop poles?

Old exam question

A control system is given by.

$$GK = \frac{K(s+3)}{s(s+1)(s+2)(s+4)}; \quad K > 0$$

Sketch the root-loci showing all your working.

Determine the value of K such that the root-loci cuts the imaginary axis. What are the corresponding closed-loop poles?

Remarks

The rules were derived from key observations.

$$GK = -1 \quad \Rightarrow \quad \begin{cases} \angle G(s) = 180^o \pm n360^o \\ \angle n(s) - \angle d(s) = 180^o \end{cases}$$

$$Kn + d = 0; \quad K = \frac{1}{|G(s)|}$$

If and only if s is on the root-loci!

These same observations will be used in semester 2 to show how ‘design’, for example placement of compensator poles and zeros, can be based on root-loci insights.

Simple gain design

Take a simple system and estimate the gain to get the best compromise between speed of response and input activity.

$$G = \frac{1}{s(s+1)^2}; \quad K$$

We will do this by hand in the lecture and then use sisotool to check our conclusions.

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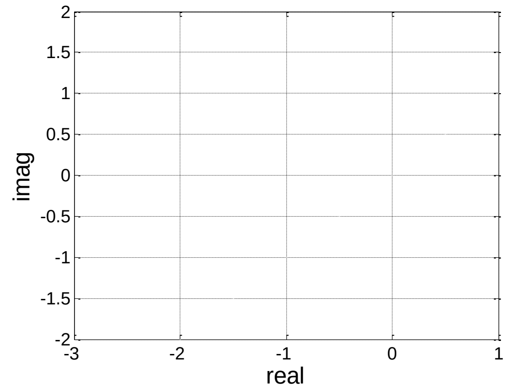

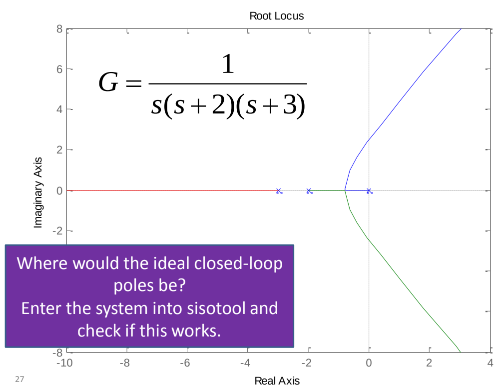

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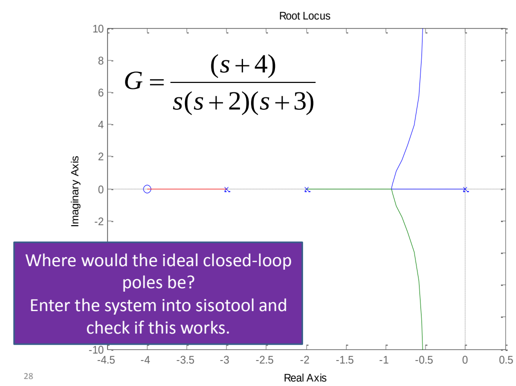

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Question

Find K to set the dominant poles for systems below as having a damping ratio of 0.7.

We will use sisotool in the lecture to observe the closed-loop responses and decide if this is a good tuning.

$$G = \frac{4}{(s+2)(s+1)}; \quad G = \frac{1}{s^3+9s^2+23s+15}$$

$$G = \frac{s+3}{(s+2)(s+1)(s+4)}; \quad G = \frac{s+4}{s(s+2)(s+3)}$$

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Lag effects gain

Compares the root-loci with and without the following two lags and comment.

$$G = \frac{s+4}{s(s+2)(s+3)};$$

$$K(s) = K\frac{s+5}{s+3}; \quad K(s) = K\frac{s+0.5}{s+0.3}$$

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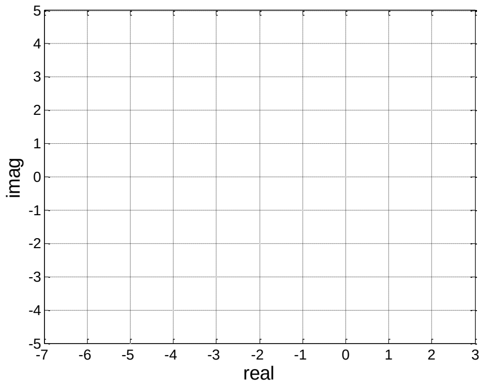
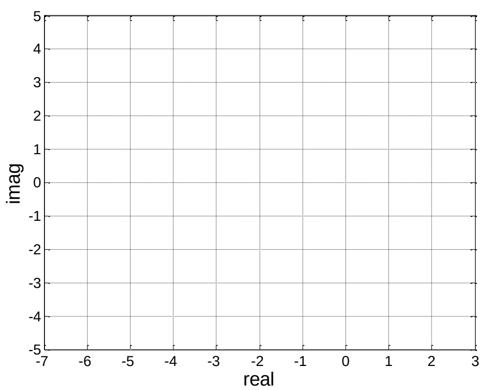


Compare root-loci with and without lead!

$$G = \frac{s+4}{s(s+2)(s+3)}; \quad K(s) = K\frac{s+3}{s+5}$$



Tutorial questions

1. Investigate the root-loci and corresponding closed-loop time responses for the following system triples.
2. Compare steady-state offsets for each case.

$$G = \frac{4}{(s+2)(s+1)}; \quad K_{lag} = K\frac{s+0.2}{s+0.05}; \quad K_{lead} = K\frac{s+1}{s+3}$$

$$G = \frac{1}{s^3+9s^2+23s+15}; \quad K_{lag} = K\frac{s+0.2}{s+0.05}; \quad K_{lead} = K\frac{s+1}{s+3}$$

$$G = \frac{s+3}{(s+2)(s+1)(s+4)}; \quad K_{lag} = K\frac{s+0.2}{s+0.05}; \quad K_{lead} = K\frac{s+1}{s+3}$$

$$G = \frac{s+4}{s(s+2)(s+3)}; \quad K_{lag} = K\frac{s+0.2}{s+0.05}; \quad K_{lead} = K\frac{s+1}{s+3}$$

Tutorial cont.

Remember, you must practise sketching root-loci by hand as well.

Use MATLAB to check your work.

1. What have you observed about lead and lag compensators ?
2. What advantages do they each bring?
3. When would you use them and why?

We will cover this in more detail in frequency response.

Questions

Use root-loci to show why one of the following systems is always closed-loop unstable and the other is always closed-loop stable, for all positive values of K.

$$G_1 = \frac{s+1}{s^2(s+2)}; \quad G_2 = \frac{s+2}{s^2(s+1)}$$

Why for examples G_1, G_3 below is there a minimum and maximum value of K to get optimum behaviour and for G_2 a maximum only?

$$G_1 = \frac{(s+3)}{s(s+1)}; \quad G_2 = \frac{s+1}{(s-1)s}; \quad G_3 = \frac{(s+1)}{s(s+0.5)(s+10)}$$