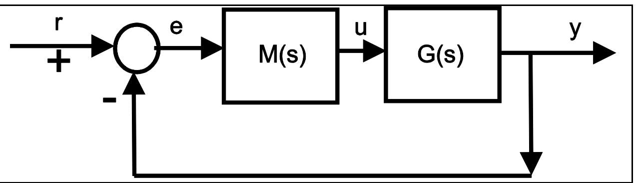
Modelling and control summaries

by Anthony Rossiter

Margins 1: Introduction

ASSUMPTION: Nyquist stability criteria gives insight into how closed-loop stability can be inferred from a Nyquist diagram of GM and knowledge of the number of open-loop RHP poles.



In addition

- **If Nyquist passed with the -1 point to the left, you expect closed-loop stability.**
- **If Nyquist passed with the -1 point to the right, you expect closed-loop instability.**

INFERENCE

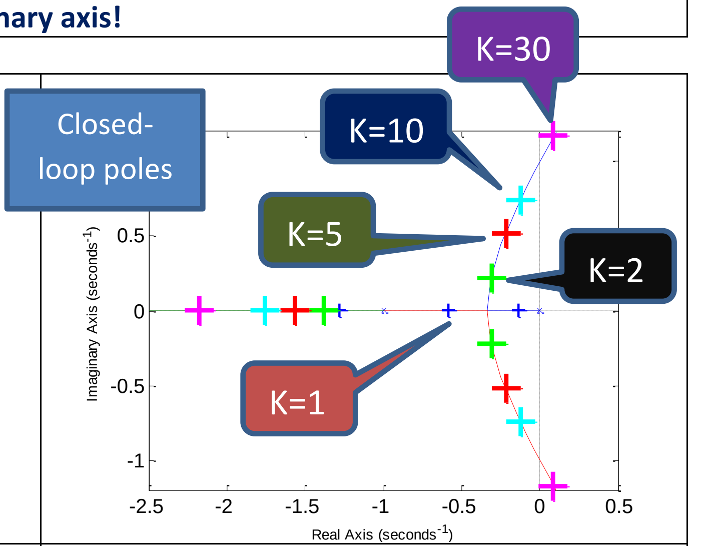
When Nyquist diagram is one side of -1 point we have LHP poles and when the other side at least one RHP pole. Assuming some form of continuity, a pole is moving from the LHP to RHP as gain changes (see root-loci). Therefore, the further the Nyquist diagram is from -1, the further we expect the dominant pole to be from the imaginary axis!

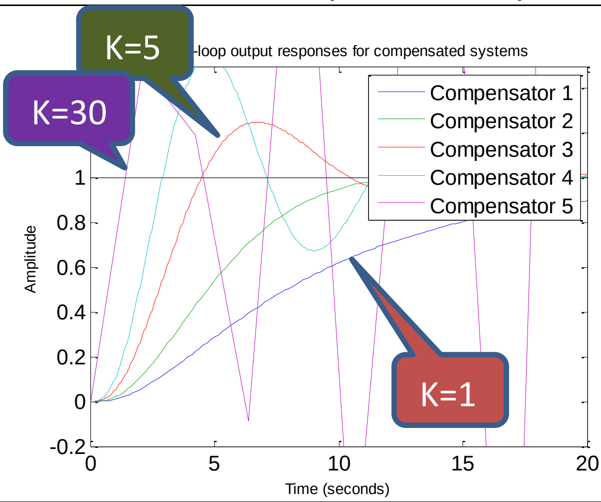
Take a simple system and watch what happens as the gain changes.

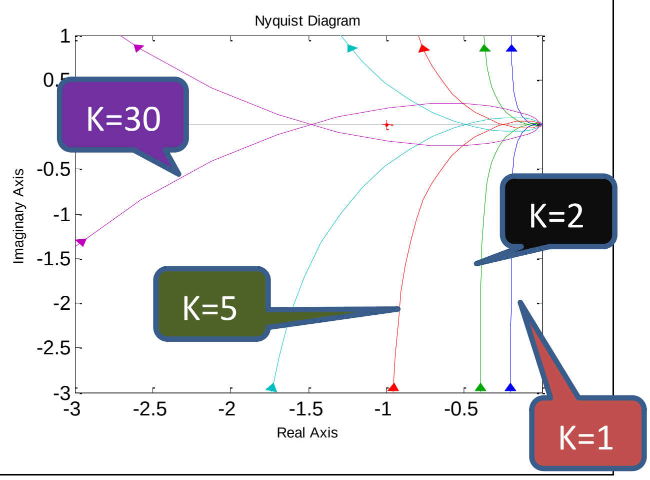
$$G = \frac{0.1K}{s(s+1)^2}$$

$$K = 1, 2, 5, 10, 30$$

As K increases to 10, Nyquist gets closer to -1, poles get closer to RHP but remain in LHP. As K>>10, Nyquist is on the other side of -1, poles move into the RHP (now unstable).







REMARKS:

Students are encouraged to try this experiment for themselves on several systems. It is probably easiest using sisotool on MATLAB as you can view the change in the closed-loop poles, Nyquist and closed-loop responses simultaneously as you change gain K.

$$G1 = \frac{(s+20)}{s(s+1)(s+4)(s+6)}; \quad G2 = \frac{(s+1)}{s^2(s+2)};$$

$$G3 = \frac{(s+4)}{(s-1)(s+2)}; \qquad M(s) = K$$

SUMMARY
"The further the –1 is to the left of the Nyquist plot, the more stable we appear to be." 1. There is no generic proof but experience shows this is generally true (up to a point). 2. Stability vs performance ? Better margins may imply low gain, slow response. 3. Small margins could imply high gain and fast response but near to instability. There is a need to find a balance.
This motivates us to seek more precision. 1. How do we define 'distance' from -1? 2. How do we define close and far? 3. How do we design and shape a Nyquist diagram to ensure good closed-loop behaviour.