Modelling and control summaries

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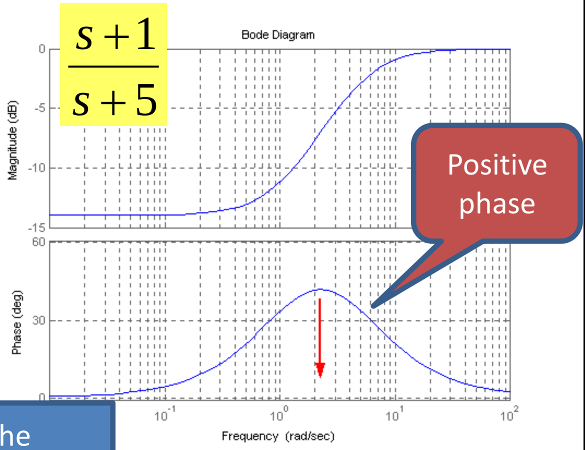
Margins 11: Simple lead compensation

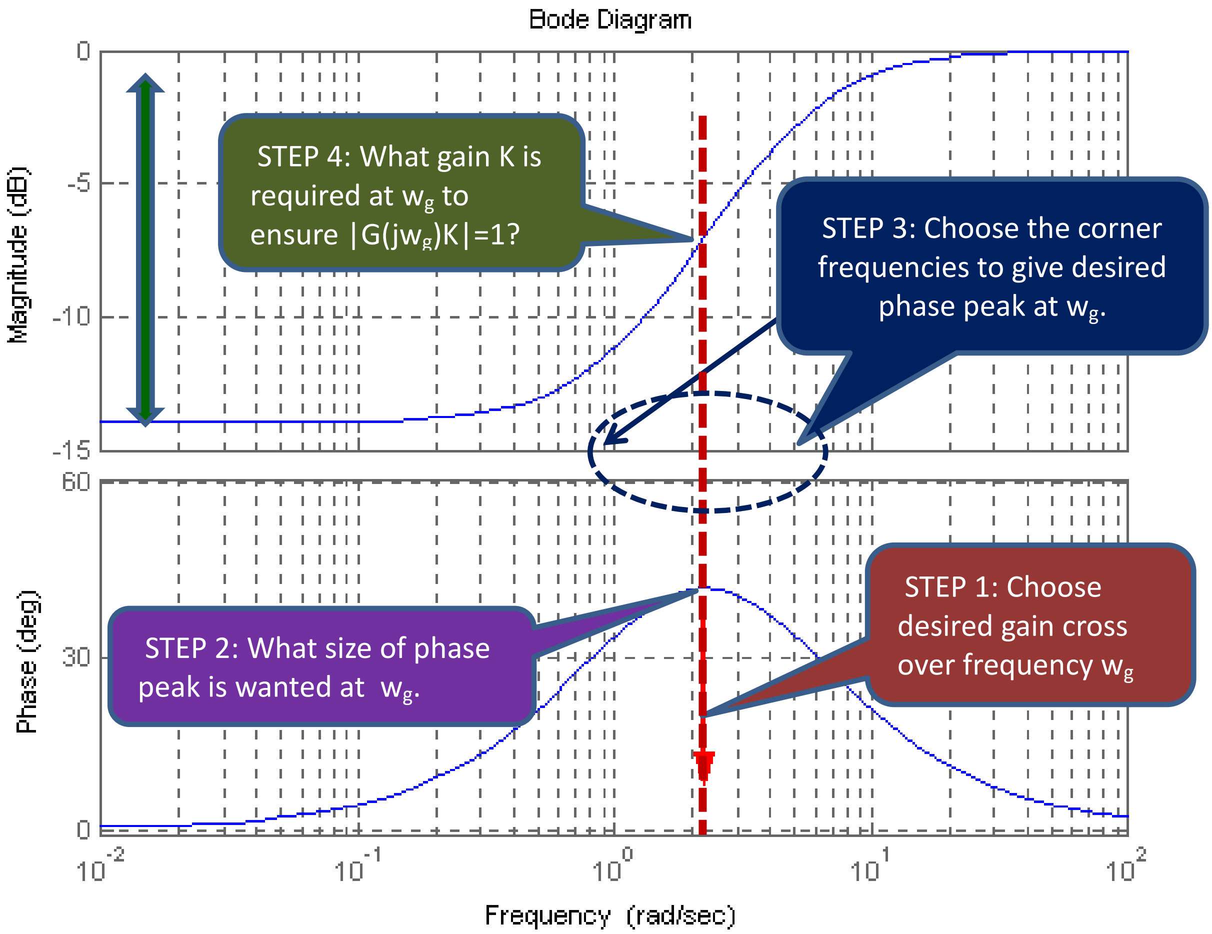
LEAD COMPENSATOR SUMMARY:

$$K\frac{s+a}{s+\beta a}; \quad 1 \le \beta \le 10$$

1. Steady state gain lower than high frequency gain (by factor of β).
2. Shifts low frequency by K/β and high freq. by K.
3. Phase is zero at high and low freq.
4. Phase is positive around corner freq.



A lead has positive phase and so can be used to rotate the Nyquist diagram away from the -1 point. This attribute allows the potential for increasing gain/bandwidth.



MECHANISTIC RULES FOR LEAD DESIGN

$$K_{lead}(s) = \frac{\sqrt{\beta}}{|G(jw)|} \frac{s + w/\sqrt{\beta}}{s + w\sqrt{\beta}}$$

1. Choose the desired gain cross-over frequency w.
2. Choose a desired PM= ϕ.
3. Find rotation ‘θ’ so that arg(G(jw)) +a =ϕ -180. Ensure θ <55!
4. Use a lookup table to find the value β for a lead with a maximum rotation ‘θ’.

β	2	3	4	6	8	10
Max. phase or θ	19	30	37	46	51	55

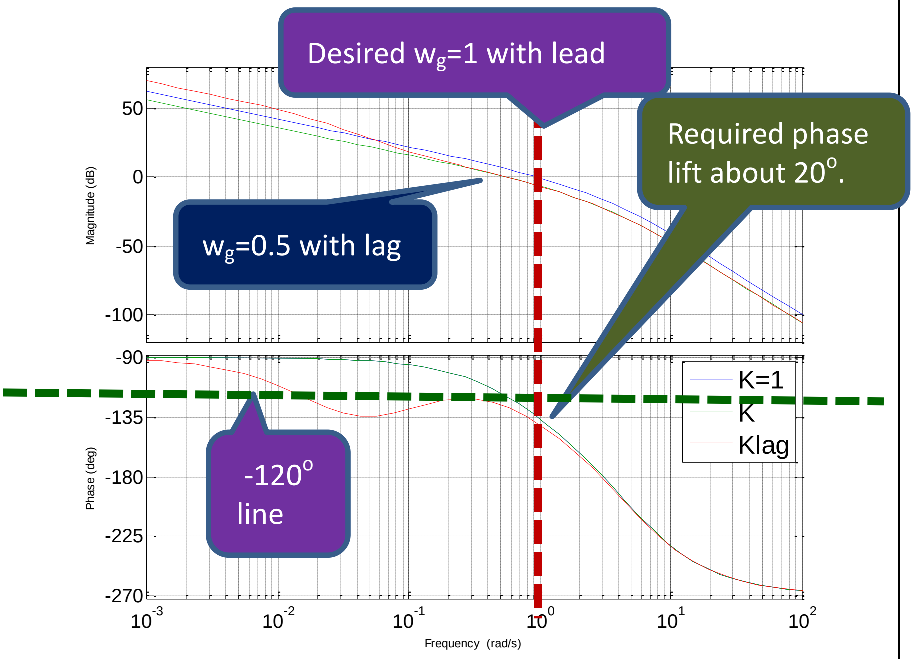
5. The control law is determined by plugging in the numbers from above.

Design a lead for G(s) to get a 60° PM given a simple gain design and arbitrary lag are already known.

$$G = \frac{10(s+2)}{s(s+1)(s+4)^2}$$

$$K = 0.5;$$

$$K_{lag} = 0.5\left(\frac{s+0.1}{s+0.02}\right)$$



Implement mechanistic design rules. Note a phase lift of 20° needs a β of about 2.

$$K_{lead}(s) = \frac{\sqrt{\beta}}{|G(jw)|} \frac{s + w/\sqrt{\beta}}{s + w\sqrt{\beta}}$$

$$w = 1, \quad \beta = 2, \quad |G(j1)| = 0.93$$

$$= \frac{\sqrt{2}}{0.93} \frac{s + 1/\sqrt{2}}{s + 1\sqrt{2}}$$