Modelling and control summaries

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Margins 12: Lead compensation tutorial

1. For the questions given here, use the mechanistic lead compensation rules to find a lead compensator.
2. Use MATLAB to compare the responses with a simple gain design, a lag design and a lead design and comment on what you find.
3. Also practice using sisotool to do the simple gain design and then to enter your lead compensator.

For simplicity, use a target phase margin of 60^o for the design of the system is open-loop stable, but you might like to investigate how behaviour varies if you use other values such as 50 or 70.

If the system is open-loop unstable, large phase margins may not be possible.

Find a lead compensator with a gain cross over frequency double what can be achieved with a simple gain design (if possible). [See margin sheet 10] What differences do you see between systems with and without integrators?	$G(s)=\frac{40}{s(s+2)};$ $G=\frac{10(s+2)}{s(s+1)(s+4)^2}$ $G(s)=\frac{0.001}{(s+0.2)(s+0.1)(s^2+0.4s+0.2)}$
Unstable systems can be hard to control effectively with a simple gain design (and therefore also with a lag). Nevertheless, one may find lead compensators are quite effective. By first choosing a sensible target phase margin and bandwidth, design lead compensators for the following.	$G=\frac{4(s+1)}{s(s+2)(s-3)}$ $G(s)=\frac{8}{(s+2)(s-0.1)}$

What are the key differences between a mechanistic lag compensator and a mechanistic lead compensator? Use evidence from the examples to back this up.

Responses can be overlaid on MATLAB using the overlaymany.m file provided on the website, e.g.:

```
>>overlaymany(G,K,Klag,Klead)
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MECHANISTIC RULES FOR LEAD DESIGN

1. Choose the desired gain cross-over frequency w. (The best choice is not always obvious but **logically it is higher** than can be achieved with a simple gain design. Often this will be a design specification.)
2. Choose a desired PM= ϕ.
3. Find rotation 'θ' so that arg(G(jw)) +a =ϕ -180. Ensure θ <55!
4. Use a lookup table to find the value β for a lead with a maximum rotation 'θ'. Typically it is good enough to round to the nearest integer as small changes have only a minor impact.

β	2	3	4	6	8	10
Max. phase or θ	19	30	37	46	51	55

5. The control law is determined by plugging in the numbers from above.

$$K_{lead}(s) = \frac{\sqrt{\beta}}{|G(jw)|} \frac{s + w/\sqrt{\beta}}{s + w\sqrt{\beta}}$$

WARNING: More advanced and systematic lead design procedures do exist. These rules are simple guidelines to give an approximate result.

POSSIBLE ANSWERS:

$$G(s) = \frac{40}{s(s+2)}; \quad K = 0.0665,\ w_g = 1.15; \quad K_{lag} = 0.0665\frac{s+0.115}{s+0.0287}$$

$$\beta = 2; \quad w_g = 2.3; \quad K_{lead} = 0.248\frac{s+1.63}{s+3.25}$$

$$G = \frac{10(s+2)}{s(s+1)(s+4)^2}; K = 0.515,\ w_g = 0.57; \quad K = 0.515\frac{s+0.057}{s+0.0142}$$

$$\beta = 2; \quad w_g = 1.14; \quad K_{lead} = 1.837\frac{s+0.806}{s+1.61}$$

$$G(s) = \frac{8}{s(s^2+3s+2)}; \quad K = 0.097; \quad w_g = 0.359; \quad K = 0.097\frac{s+0.0359}{s+0.00898}$$

$$\beta = 3; \quad w_g = 0.72; \quad K_{lead} = 0.4067\frac{s+0.415}{s+1.24}$$

Cannot double frequency so instead use maximum β=10 and see what increase is possible.

$$G(s) = \frac{0.001}{(s+0.2)(s+0.1)(s^2+0.4s+0.2)}; \quad K = 9.35; \quad w_g = 0.167; \quad K_{lag} = 9.35\frac{s+0.0167}{s+0.00417}$$

$$\beta = 10; \quad w_g = 0.3; \quad K_{lead} = 58.7\frac{s+0.0949}{s+0.949}$$