Modelling and control summaries

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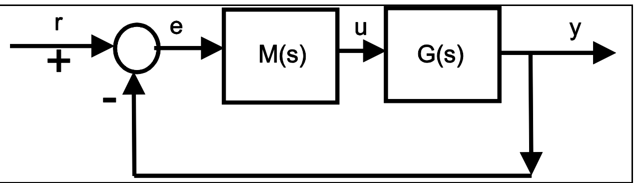
Margins 2: gain margin

ASSUMPTION: The system is closed-loop stable and the Nyquist Diagram passes with the -1 point to the left.



HOW MUCH CAN I CHANGE GAIN BEFORE THE SYSTEM GOES CLOSED-LOOP UNSTABLE? The mutiplicative change is called the gain margin.

In order for gain margin to exist, the Nyquist diagram must cross the negative real axis so that changing gain can change the number of encirclements of -1 which implies there must exist w such that **arg(G(jw))=-180°**.

1. Find a real number K such that G(jw)K=-1, then Gain margin =K.
2. Alternatively find the intercept 'a' with the negative real axis: GM=1/|a|.

ILLUSTRATION

$$G(s)M(s) = \frac{K}{s(s+1)^2}$$

It is clear that if K=2, then the Nyquist diagram will pass through the -1 point.

GAIN MARGIN is 2

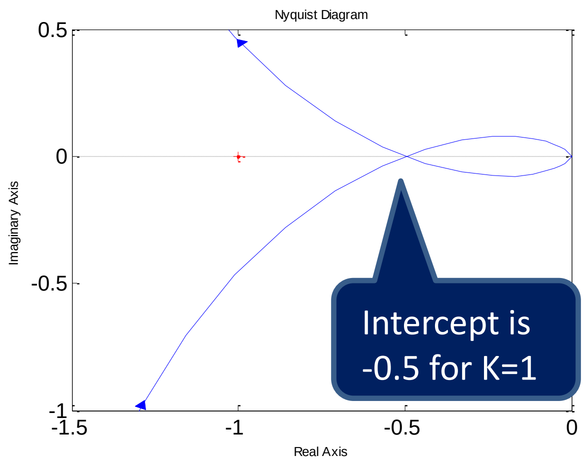
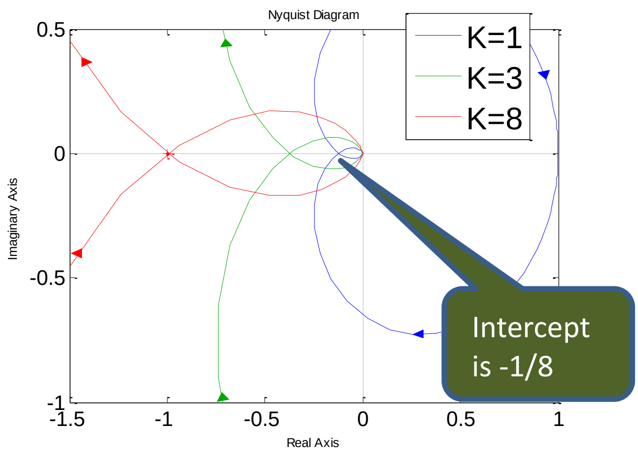


ILLUSTRATION 2

$$G(s)M(s) = \frac{K}{(s+1)^3}$$

It is clear that if K=8, then the Nyquist diagram will pass through the -1 point.

GAIN MARGIN is 8



PROCEDURE FOR FINDING THE GAIN MARGIN

Gain margin is computed where the Nyquist diagram crosses the negative real axis, therefore when **arg(G(jw))=-180°** .

1. Find phase cross over frequency w_p where $\arg(G(jw_p))=-180^o$.
2. Find $|G(jw_p)|$ at this frequency.
3. The intercept with the negative real axis must be at $-|G(jw_p)|$. **GM is given as $K=1/|G(jw_p)|$.**

EXAMPLE 1 $$G=\frac{1}{s(s+1)(s+2)}$$	$1.\left\{\angle G=-90-\tan^{-1}w-\tan^{-1}\frac{w}{2}=-180\right\} \Rightarrow \tan^{-1}w+\tan^{-1}\frac{w}{2}=90$ $HENCE \quad \frac{w^2}{2}=1 \quad or \quad w_p=\sqrt{2}$ $2.\left|G(jw_p)\right|=\frac{1}{\sqrt{2(2+1)(2+4)}}=\frac{1}{6} \Rightarrow 3.\,GM=6$
EXAMPLE 2 $$G=\frac{1}{s(s+3)(s+4)}$$	$1.\left\{\angle G=-90-\tan^{-1}\frac{w}{3}-\tan^{-1}\frac{w}{4}=-180\right\} \Rightarrow \tan^{-1}\frac{w}{3}+\tan^{-1}\frac{w}{4}=90$ $HENCE \quad \frac{w^2}{12}=1 \quad or \quad w_p=\sqrt{12}$ $2.\left|G(jw_p)\right|=\frac{1}{\sqrt{12(12+9)(12+16)}}=\frac{1}{84} \Rightarrow 3.\,GM=84$
EXAMPLE 3 $$G=\frac{2}{s(s+3)}$$	$1.\left\{\angle G=-90-\tan^{-1}\frac{w}{3}=-180\right\} \Rightarrow \tan^{-1}\frac{w}{3}=90$ $HENCE \quad w\to\infty$ NO INTERCEPT with negative real axis and hence GM cannot be defined!

REMARKS:

1. Approx. guidelines are: Gain Margin should be at least 3.
2. This is not a design target however, but simply a minimum suggestion!
3. Gain margin may be infinite and still the closed-loop has bad behaviour.
4. However small GM, that is below 3, will nearly always result in unsatisfactory behaviour.

PHASE CROSS OVER FREQUENCY

This is the frequency where the Nyquist diagram crosses the negative real axis.

Phase cross over frequency is use to compute **gain** margin.

PROBLEMS: Find the gain margins for the following

$$G=\frac{6}{s(s+0.2)}; \quad G=\frac{0.05}{s(s+0.1)(s+0.4)}; \quad G=\frac{(s+2)}{s(s+1)}; \quad G=\frac{6(s+1)}{s^2(s+4)^2};$$