Modelling and control summaries

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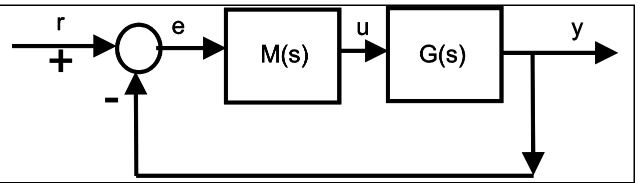
Margins 3: phase margin

ASSUMPTION: The system is closed-loop stable and the Nyquist Diagram passes with the -1 point to the left.



HOW MUCH CAN I CHANGE PHASE BEFORE THE SYSTEM GOES CLOSED-LOOP UNSTABLE? The clockwise rotational change is called the phase margin.

In order for phase margin to exist, the Nyquist diagram must have a gain greater than unity so that changing phase can change the number of encirclements of -1 which implies there must exist w such that **|G(jw)|>1**.

1. Find a clockwise rotation $e^{-j\phi}$ such that $G(jw)\, e^{-j\phi} = -1$.
2. Note this implies $\arg(G(jw)) - \phi = -180^o$.
3. Phase margin $= \phi = 180 + \arg(G(jw))$.

Use of minus ϕ as clockwise is a negative rotation.

ILLUSTRATION

$$G(s)M(s) = \frac{1}{s(s+1)^2}$$

If rotate Nyquist clockwise by 21^o then it will pass through the -1 point.

PHASE MARGIN is $21^o = 180 + \arg(G(jw))$.

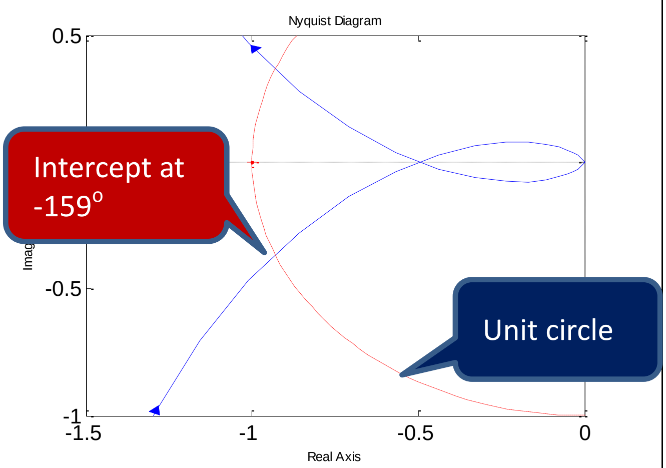
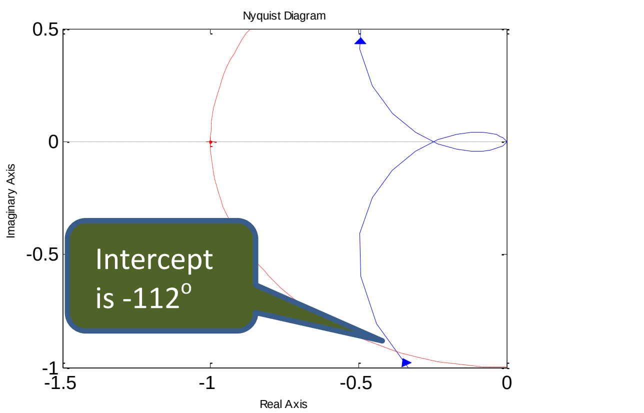


ILLUSTRATION 2

$$G(s)M(s) = \frac{2}{(s+1)^3}$$

If rotate Nyquist clockwise by 68^o then it will pass through the -1 point.

PHASE MARGIN is $68^o = 180 + \arg(G(jw))$.



PROCEDURE FOR FINDING THE PHASE MARGIN

Phase margin is computed where the Nyquist diagram crosses the unit circle, therefore.

1. Find gain cross over frequency w_g where $|G(jw_g)|=1$.
2. Find $\arg(G(jw_g))$ at this frequency.
3. The clockwise phase rotation required, or phase margin = $180 + \arg(G(jw_g))$.

NOTE: For most real systems, the PM cannot be computed analytically – numerical approaches must be used.

EXAMPLE 1 $G = \dfrac{1}{s(s+0.2)}$	$1.\left\{|G(jw)| = \dfrac{1}{\sqrt{w^2(0.2^2+w^2)}} = 1\right\} \Rightarrow w^4 + 0.2^2 w^2 - 1 = 0$ $2.\left\{w_g = 0.99 \Rightarrow \angle G = -90 - \tan^{-1}\dfrac{w_g}{0.2} = -168.6\right\}$ $HENCE \quad 3. \; PM = 180 - 168.6 = 11.4$
EXAMPLE 2 $G = \dfrac{1}{(s+1)(s+2)}$	$1.\left\{|G(jw)| = \dfrac{10}{\sqrt{(w^2+1)(2^2+w^2)}} = 1\right\} \Rightarrow w^4 + 5w^2 - 95 = 0$ $2.\left\{w_g = 2.75 \Rightarrow \angle G = -\tan^{-1}\dfrac{w_g}{1} - \tan^{-1}\dfrac{w_g}{2} = -124\right\}$ $HENCE \quad 3. \; PM = 180 - 124 = 56$

REMARKS: Approx. guidelines are

1. Phase Margin should be around 60^o.
2. This is often a design target which means:
   - Smaller margins may imply poor behaviour.
   - Larger margins may imply the system is slow, or too detuned.
   - Nevertheless, thus use of 60^o is somewhat arbitrary so students should be flexible. Perhaps 45^o will be enough, or maybe you need 70^o.
3. **Implies that the Nyquist diagram crosses the unit circle in quadrant 3!**

GAIN CROSS OVER FREQUENCY

This is the frequency where the Nyquist diagram crosses the unit circle.

GAIN cross over frequency is use to compute **PHASE** margin.

PROBLEMS: Find the phase margins for the following – you will find they are not all possible with simple paper and pen computations.

$$G = \frac{6}{s(s+0.4)}; \quad G = \frac{0.05}{s(s+0.1)(s+0.4)}; \quad G = \frac{(s+2)}{s(s+1)}; \quad G = \frac{6(s+1)}{s^2(s+4)^2};$$