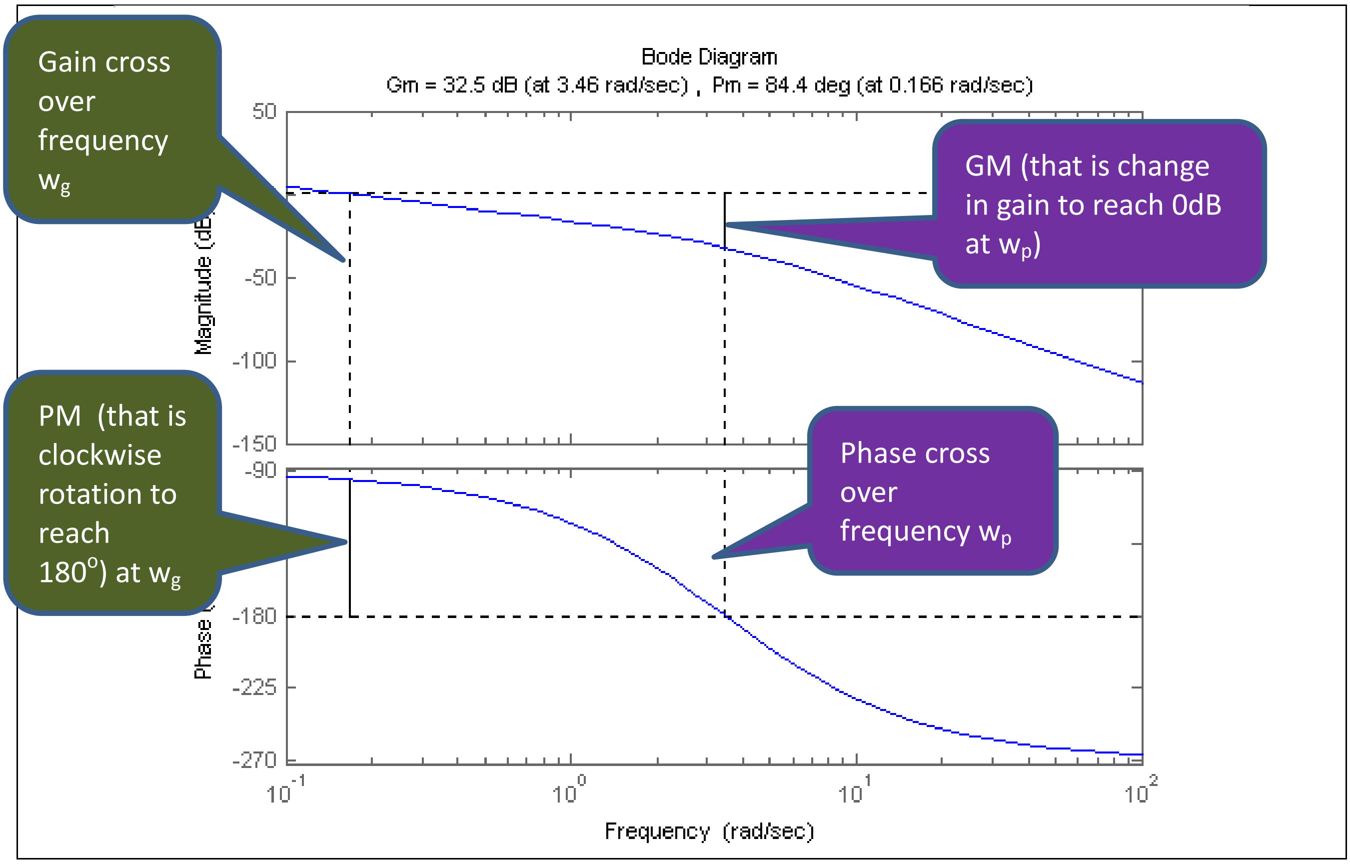
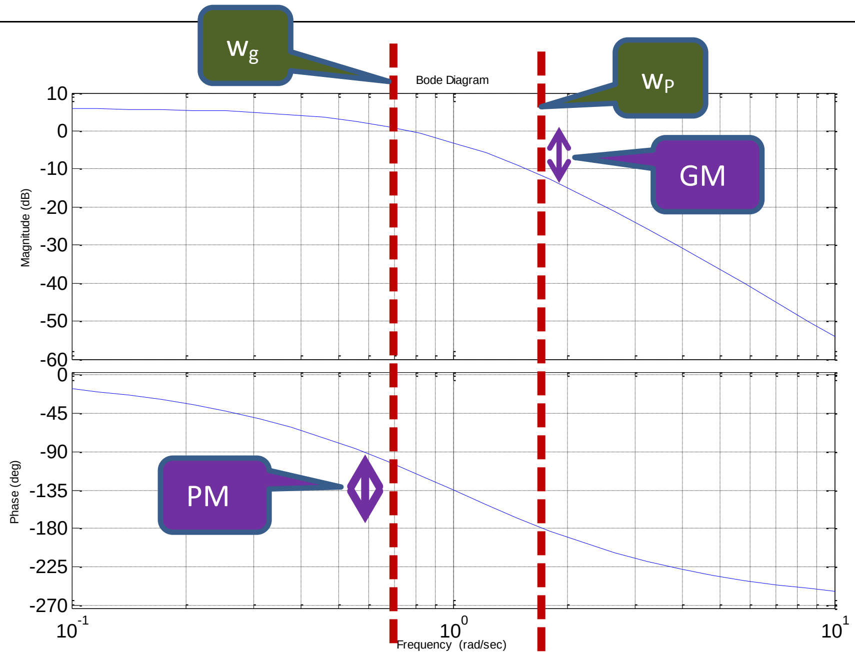
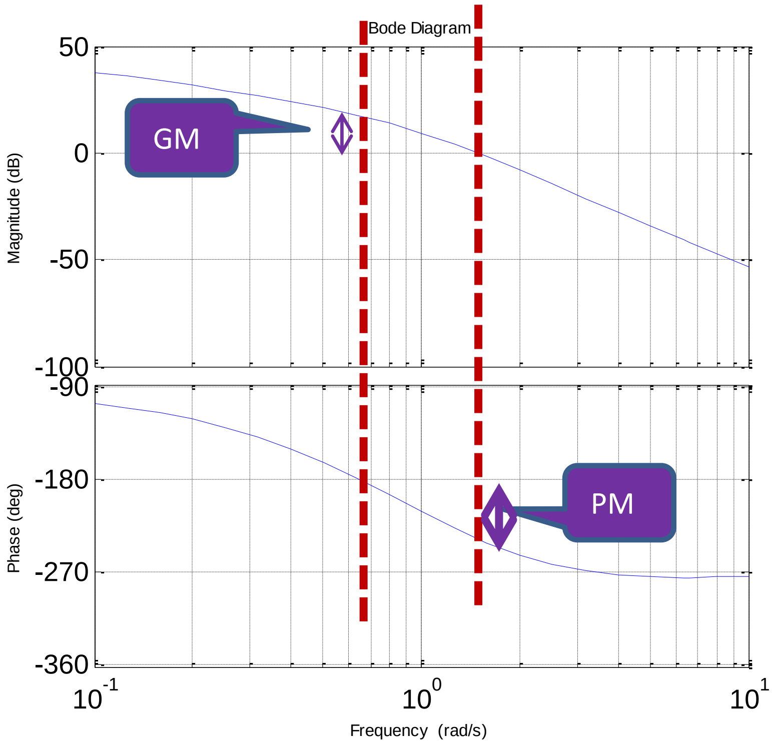
Modelling and control summaries

by Anthony Rossiter

Margins 4: Using Bode diagrams

<table>
<tr><td>PHASE MARGIN:
1. Find a frequency w_g such that $|G(jw_g)|=1$.
2. Find a clockwise rotation $e^{-j\phi}$ such that $G(jw_g)\, e^{-j\phi} = -1$.
3. Phase margin $= \phi = 180+\arg(G(jw_g))$.</td><td>GAIN MARGIN
1. Find w_p such that $\arg(G(jw_p))=-180$.
2. Find a real number K such that $G(jw_p)K=-1$.
3. Gain margin $= K = 1/|G(jw_p)|$</td></tr>
<tr><td colspan="2">The definitions of gain and phase margins are clear, but in general the algebra is too demanding and not insightful. Can we use Bode diagrams to simplfy the computation of margins and give some insight?</td></tr>
<tr><td colspan="2">REMARKS for MARGINS in a Bode diagram:
Easier to represent GM using decibels and hence GM (in dB)$=20\log_{10}(1/|G(jw_p)|)=-20\log_{10}(|G(jw_p)|)$
GM (in dB) >0 (+ve gain margin) corresponds to a scaling of the Nyquist greater than 1.
Positive phase margin is read in a clockwise direction, so the desired gain cross over frequency would be where the Nyquist diagram is in quadrant 3 (between -90^o and -180^o).
Gain cross over frequency corresponds to the 0dB line.</td></tr>
</table>



EXAMPLE 1 OF PROCESS Draw vertical lines at the gain and phase cross over frequencies. Positive PM is distance above -180 at w_g Positive GM is distance below 0dB at w_p.	
EXAMPLE 2 Margins can be negative. Here the gain plot is above 0dB at the phase cross over frequency and the phase plot is below -180 at the gain cross over frequency. Negative margins will normal imply closed-loop instability.	

ALTERNATIVES WAYS OF USING MATLAB:

1. Create the bode diagram using bode.m and estimate the margins by eye.
2. Create the bode diagram with margins superimposed using margin.m .
3. Enter system into sisotool. Margins displayed automatically in bode diagram.
4. Use MATLAB to find the margins for the following.

$$G=\frac{6}{s(s+0.4)};\quad G=\frac{0.05}{s(s+0.1)(s+0.4)};\quad G=\frac{(s+2)}{s(s+1)};\quad G=\frac{6(s+1)}{s^2(s+4)^2};$$