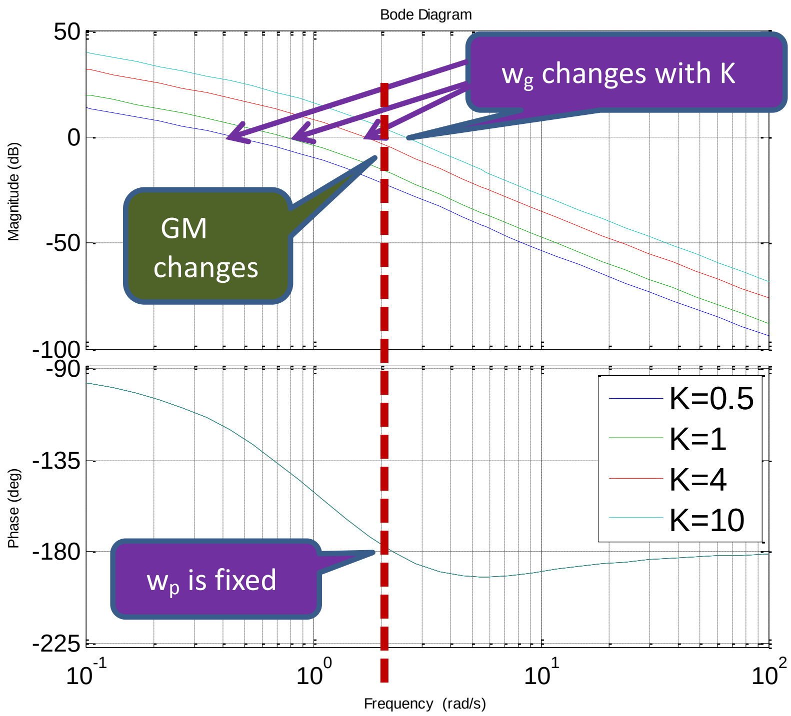
Modelling and control summaries

by Anthony Rossiter

Margins 6: Effects of gain changes on GM

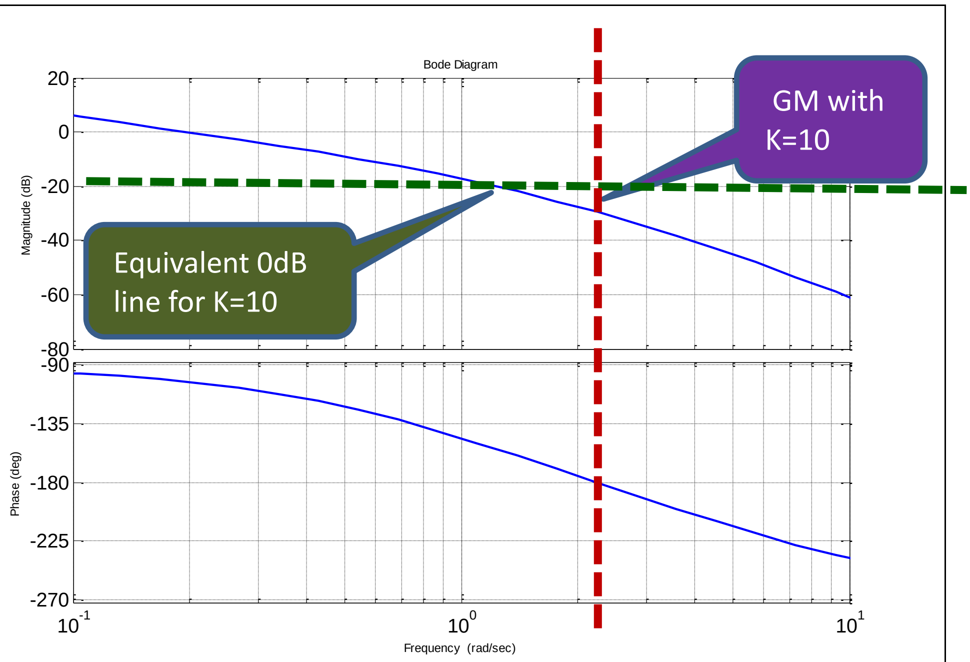
PHASE MARGIN:	**GAIN MARGIN**
1. Find a frequency w_g such that $\|G(jw_g)\|=1$. 2. Clockwise rotation $e^{-j\phi}$ such that $G(jw_g)\, e^{-j\phi}=-1$. 3. Phase margin $=\phi = 180+\arg(G(jw_g))$.	1. Find w_p such that $\arg(G(jw_p))=-180$. 2. Find a real number K such that $G(jw_p)K=-1$. 3. Gain margin = $K = 1/\|G(jw_p)\|$

EXAMPLE $G(s)=\frac{0.4}{(s+2)(s+1)}$ $M(s)=K\frac{s+5}{s}$ Changes in gain cause the gain plot to move up and down, but have no impact on the phase plot. **The gain crossover frequency changes.** **The phase crossover frequency is fixed.**	
IMPACT ON GAIN MARGIN As w_p is fixed, the gain margin is computed at a fixed frequency. $GM=-20\log_{10}\|G(jw_p)K\|$	Gain margin changes as **$-20\log_{10}(K)$** K=1, Gain margin =17.5dB $(=-20\log10\|G(jw_p)\|)$ K=0.5, Gain margin $=17.5-20\log_{10}(0.5)=23.5$ K=4, Gain margin $=17.5-20\log_{10}(4)=5.5$ K=10, Gain margin $=17.5-20\log_{10}(10)=-2$
What K will give a GM of 20dB?	$GM=-20\log10\|G(jw_p)K\|=-20\log10\|G(jw_p)\|-20\log_{10}(K)$ $20 = 17.5-20\log_{10}(K)$ or $K = 10^{(-0.125)}$

Given the Bode diagram with K=1, find the gain margin for K=10, 100

A use of K=10 is will move the gain diagram up by 20dB. Equivalently, one could compute the gain margin on the -20dB line.

A use of K=100 can be represented by considering the -40dB line as equivalent to 0dB.

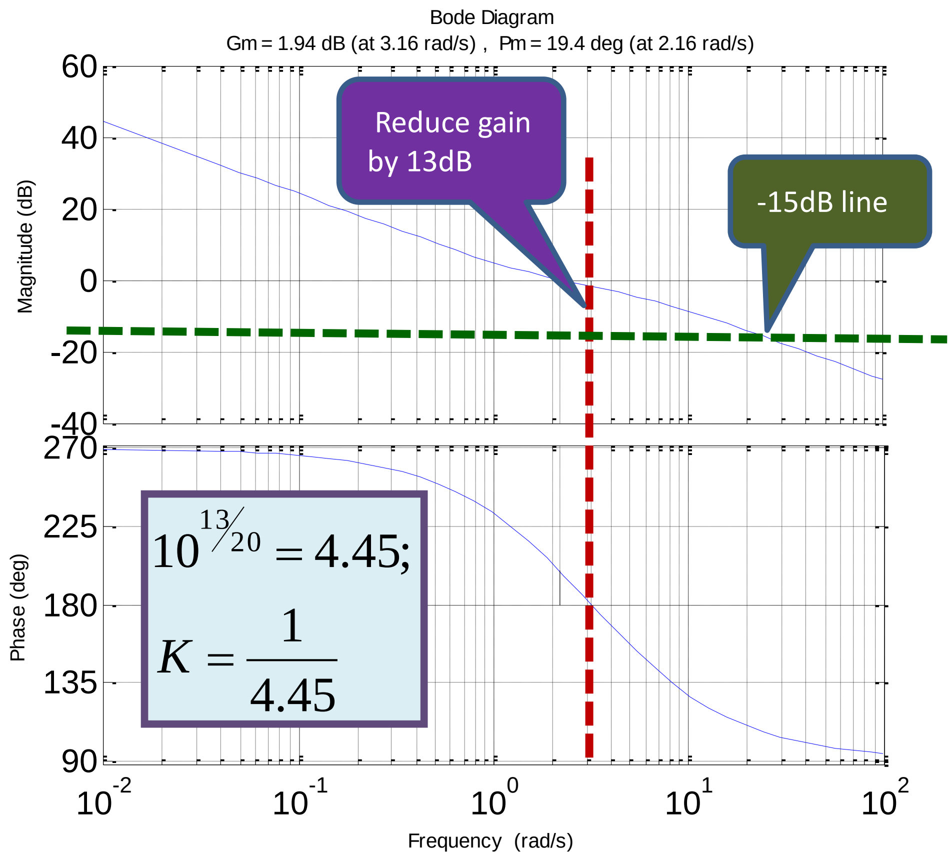


Find K such that the gain margin is 15dB given the diagram is for K=1.

1. Draw the vertical line to represent the phase cross over frequency.

2. Need gain plot to intercept this line at -15dB in order to get a 15dB gain margin.

3. Move gain plot down to ensure intercept. The amount is obvious from the plot.



SUMMARY: Rather than redrawing the gain plot for different values of K, it is often easier to recognise that introducing K is equivalent to moving the 0dB line by $20\log_{10}(K)$.