

# Modelling and control summaries



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## Margins 7: Effects of gain changes on PM

### PHASE MARGIN:

1. Find a frequency  $\omega_g$  such that  $|G(j\omega_g)|=1$ .
2. Clockwise rotation  $e^{-j\phi}$  such that  $G(j\omega_g) e^{-j\phi} = -1$ .
3. Phase margin =  $\phi = 180 + \arg(G(j\omega_g))$ .

### GAIN MARGIN

1. Find  $\omega_p$  such that  $\arg(G(j\omega_p)) = -180$ .
2. Find a real number  $K$  such that  $G(j\omega_p)K = -1$ .
3. Gain margin =  $K = 1/|G(j\omega_p)|$

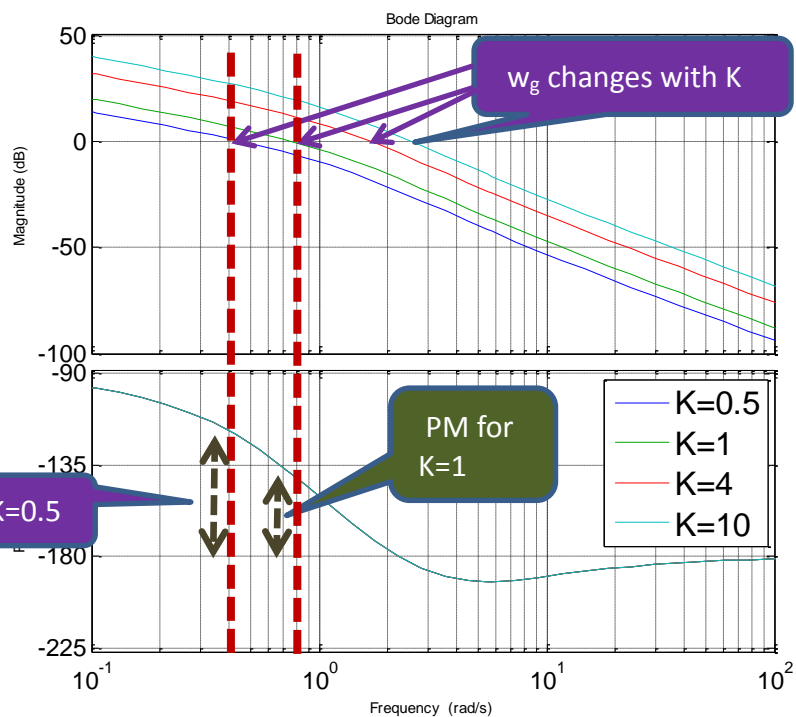
### EXAMPLE

$$G(s) = \frac{0.4}{(s+2)(s+1)}$$

$$M(s) = K \frac{s+5}{s}$$

Changes in gain cause the gain plot to move up and down, so **the gain crossover frequency changes.**

**Consequently, the PM is computed at a different point and changes.**



In general, there is no simple **analytic formulae** which captures the change in the PM with  $K$ .

**This is because the gain crossover frequency does not have a simple analytic dependence on  $K$  (in general).**

A new computation is required for each value of  $K$ .

**NEXT, DESPITE THE LACK OF EASY ANALYTIC RESULTS, WE SHOW HOW BODE DIAGRAMS CAN BE USED QUICKLY AND EFFICIENTLY TO ACHIEVE A SPECIFIED PHASE MARGIN BY JUST CHANGING THE GAIN.**

## ACQUIRING A SPECIFIED PHASE MARGIN

The phase margin is computed at the gain cross over frequency using  $\phi = 180 + \arg(G(j\omega_g))$ . Alternatively, a specified phase margin will dictate the gain cross over frequency indirectly by specifying the phase of  $G(j\omega)$  where the gain cross over frequency must occur:

**STEP 1: Find required  $\omega_g$  so that  $\arg(G(j\omega_g))$  has desired value ( $\phi - 180 = \arg(G(j\omega_g))$ ).**

**STEP 2: Find K such that  $|G(j\omega_g)K|=1$  – that is force  $\omega_g$  to be gain cross over frequency.**

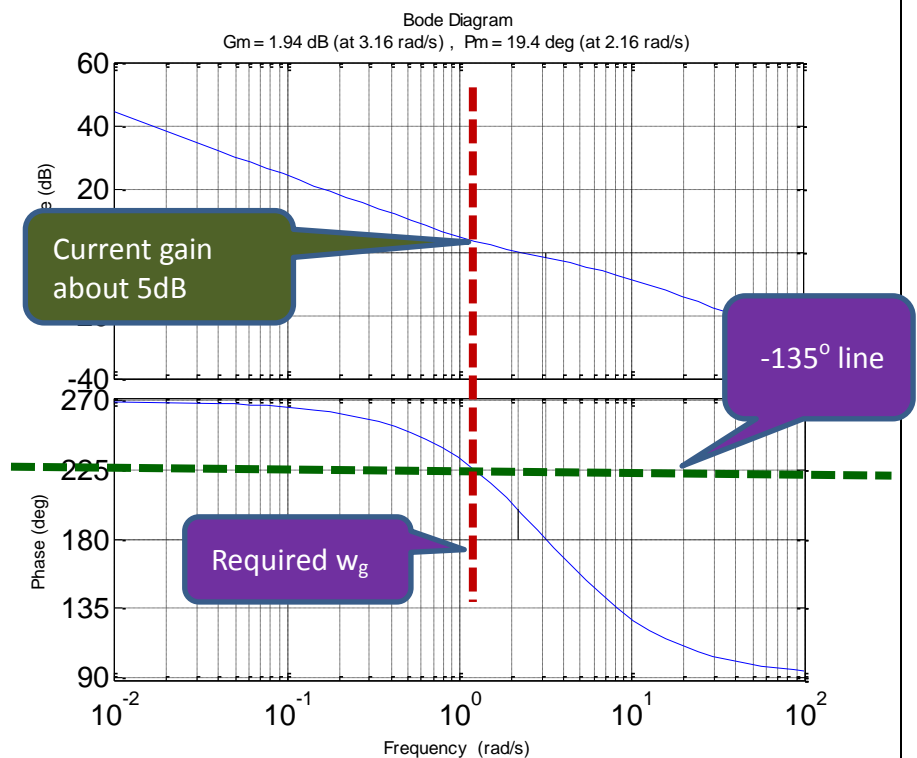
**Find K so that the phase margin is 45 degrees.**

STEP 1: Find the frequency where  $\arg(G(j\omega)) = -180 + 45 = -135$

STEP 2: Draw a vertical line up at this frequency to find the current gain.

Use  $K = 1/|G(j\omega)|$  in order to force this to become the gain cross over frequency – that is make  $|G(j\omega)K|=1$ .

Here K will be -5dB



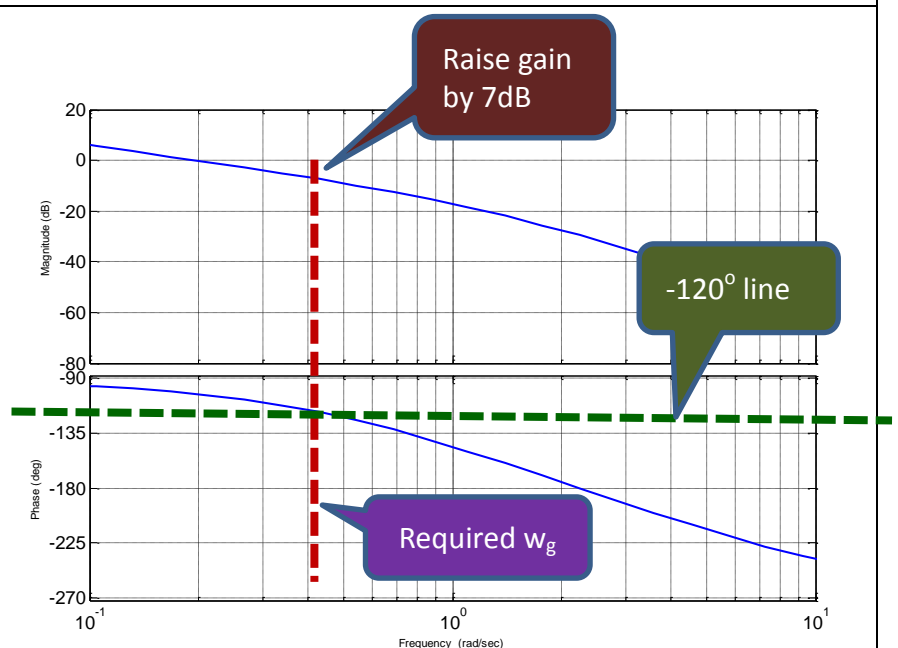
**Find K to achieve a 60 degree phase margin**

1. STEP 1: Find the frequency where  $\arg(G(j\omega)) = -180 + 60 = -120$

STEP 2: Draw a vertical line up at this frequency to find the current gain.

Find the gain change required to make  $|G(j\omega)K|=1$ .

Here K will be about 7dB



**Warning:** Not all phase margins are possible – only those where we can satisfy  $\phi - 180 = \arg(G(j\omega_g))$ .