

# Modelling and control summaries



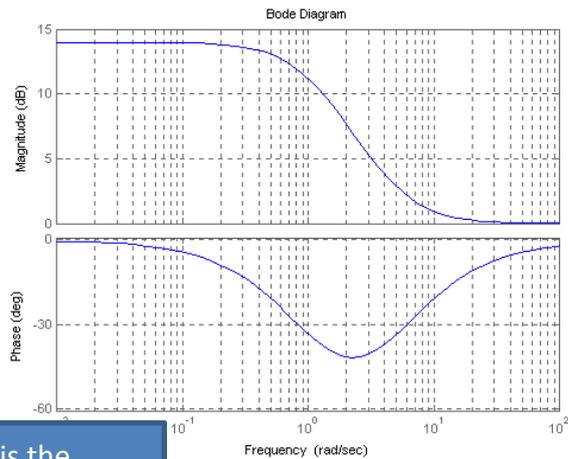
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## Margins 9: Simple lag compensation

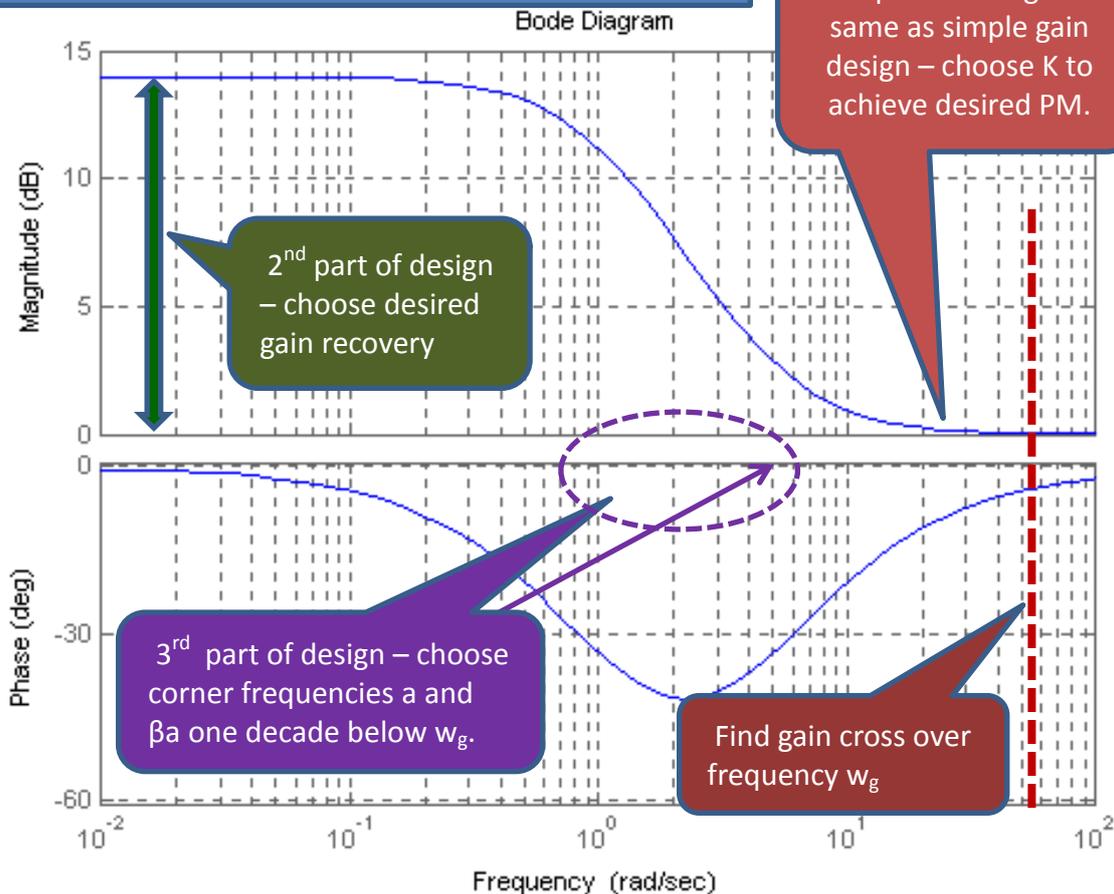
### LAG COMPENSATOR SUMMARY:

$$K \frac{s + \beta a}{s + a}; \quad 1 \leq \beta \leq 10$$

1. Steady state gain larger than high frequency gain (by factor of  $\beta$ ).
2. Shifts low frequency up by  $K\beta$  and high freq. by  $K$ .
3. Phase is zero at high and low freq.
4. Phase is negative around corner freq.



The difference between a simple  $K$  design and a lag is the ability to recover gain at low frequencies. Otherwise expect equivalent bandwidth and margins.



For  $G=K/(s+1)^4$ , use a phase lag which gives a PM=60 and recovers a factor of 5 gain at low frequency.

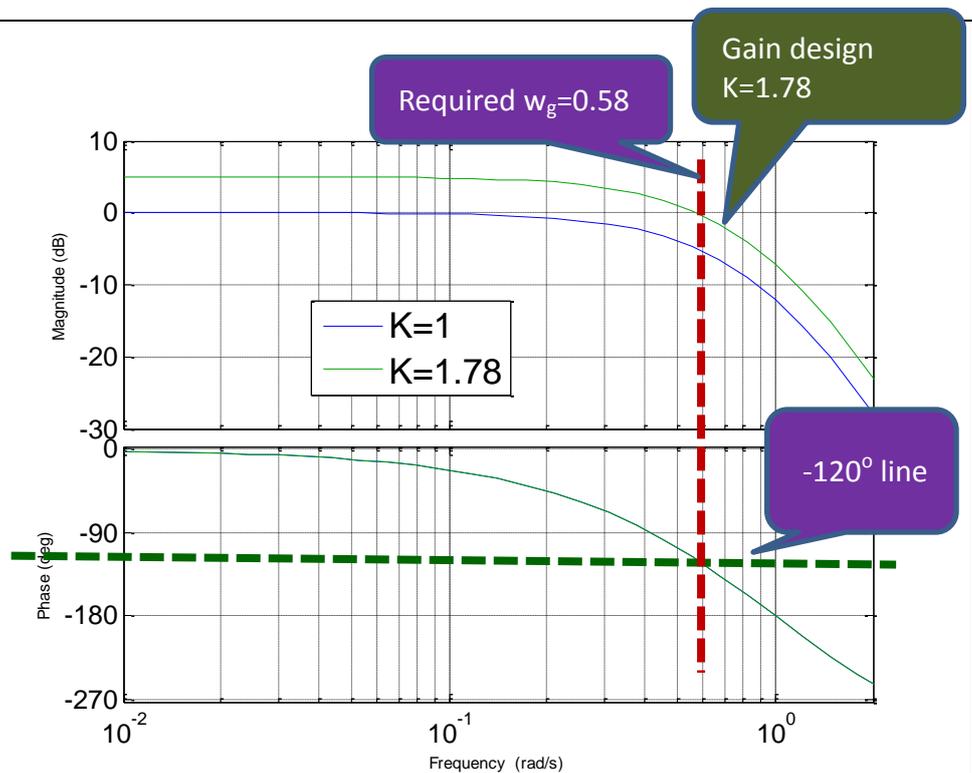
**STEP 1:** Do a simple gain design to get a 60° PM.

**STEP 2:** Determine  $w_g$ .

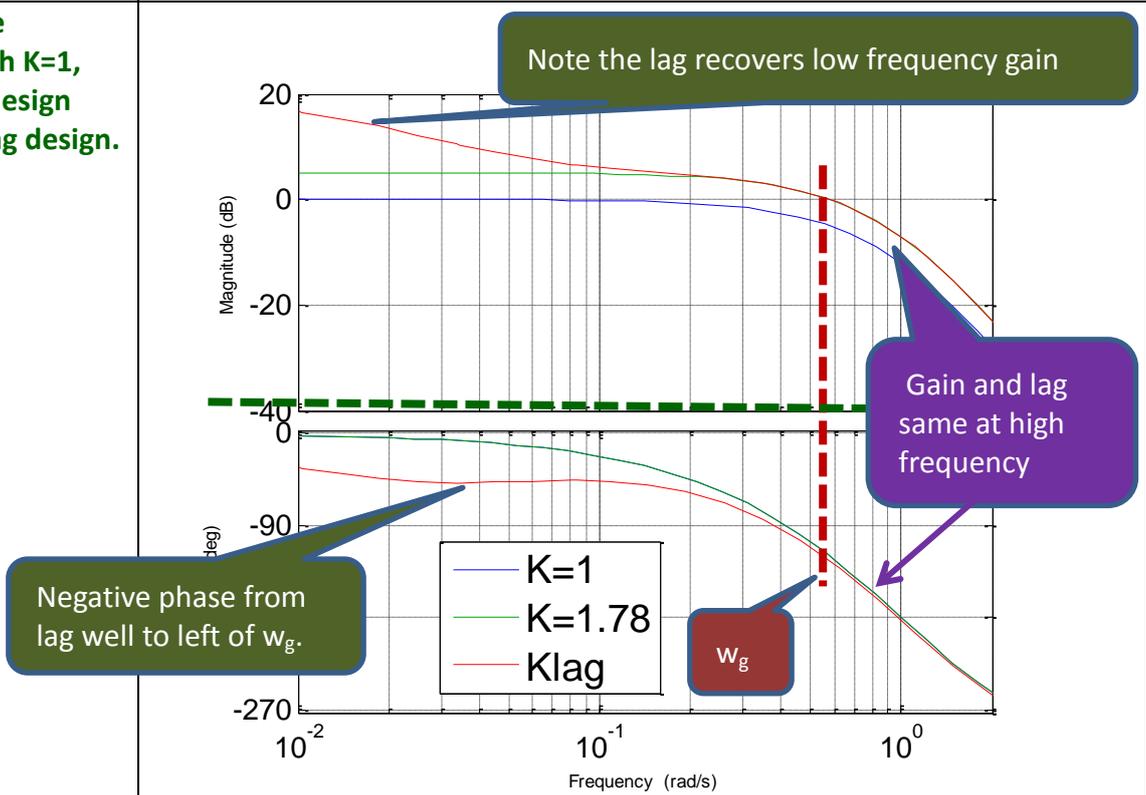
**STEP 3:** Define lag using specified K,  $w_g$  and gain recovery  $\beta=5$ .

$$K_{lag}(s) = K \frac{s + \frac{w_g}{10}}{s + \frac{w_g}{10\beta}}$$

$$K_{lag} = 1.78 \frac{s + 0.058}{s + 0.0116}$$



**Overlay Bode diagrams with K=1, simple gain design and simple lag design.**



1. A lag is only useful if there exists a suitable simple gain design to begin with!
2. The lag introduces negative phase which gives a small loss in PM compared to the simple gain design, but increases low frequency gain by  $\beta$ .
3. Implicit that the corner frequencies must be at least a decade below gain cross over frequency to avoid the negative phase having a significant impact on margins.
4. Low gain design as improves margins by gain reduction at high frequency.