

Modelling and control summaries

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Nyquist 11: Concepts of encirclements

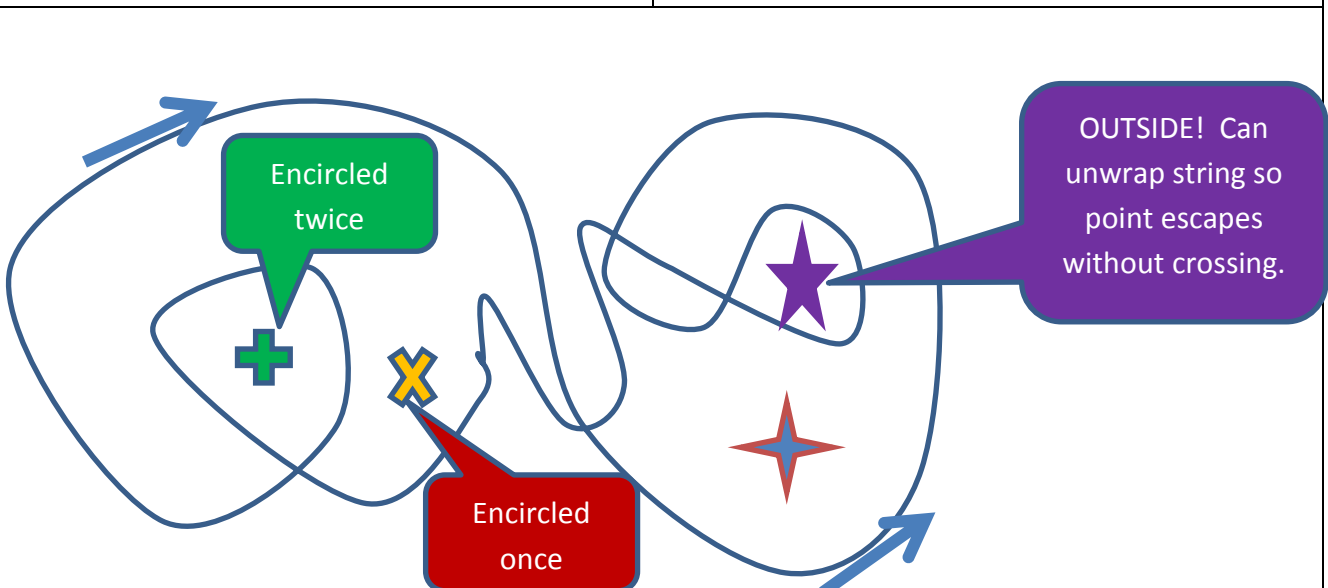
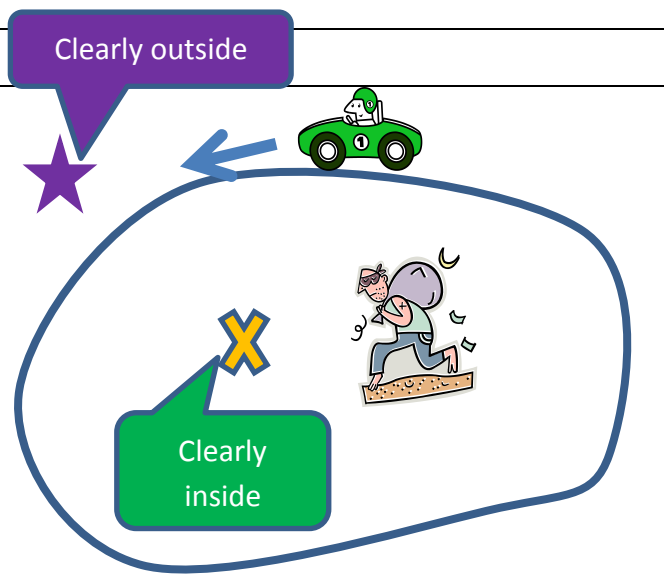
The full definition of a Nyquist diagram is the mapping of $G(s)$ while s describes the D-contour.

We need to decide whether a given point (usually -1) lies inside or outside of a Nyquist diagram. If inside, how many times does the diagram go around?

Simple illustration

Think of the Nyquist diagram like a string. Can a given point escape without cutting the string? This allows the user to move and bend the string however they wish.

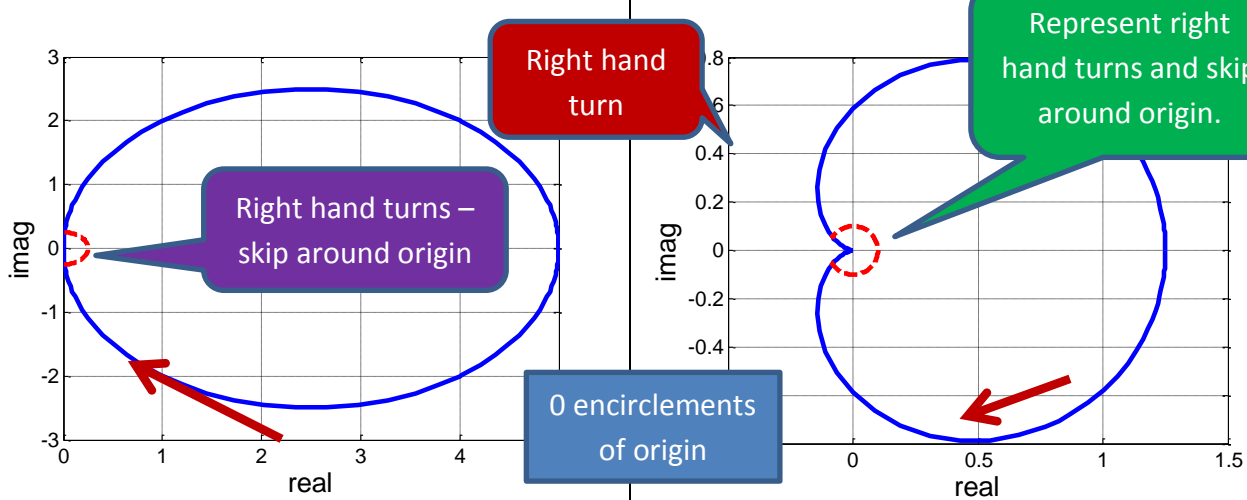
Number of encirclements is number of times the point needs to cross the string to escape.



NEXT: Look at some examples of Nyquist diagrams and count the encirclements of the **ORIGIN** and dependence on pole/zero positions.

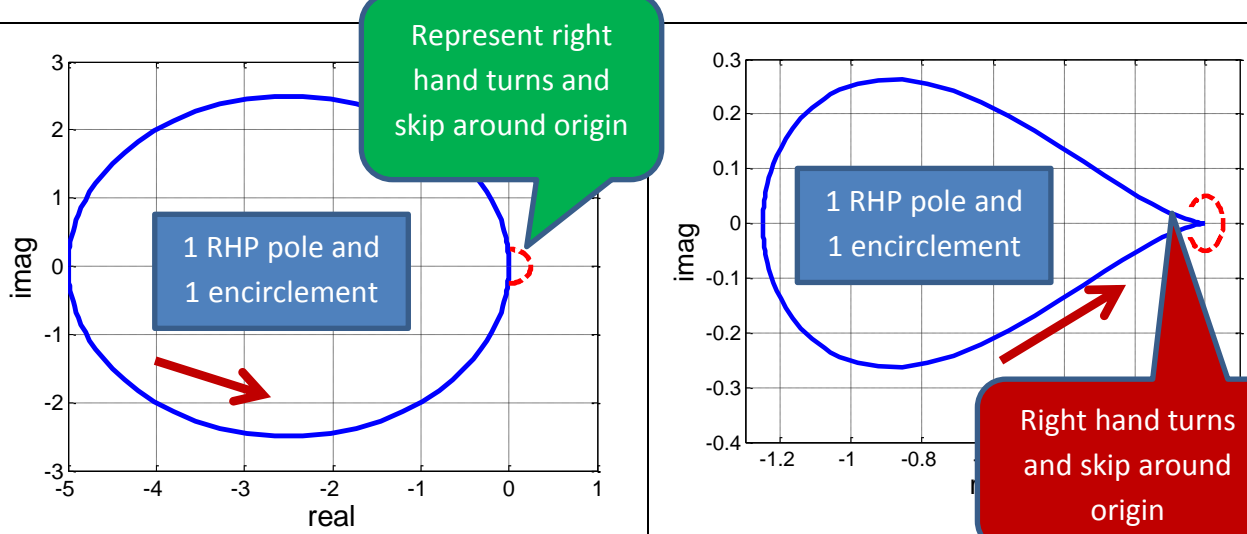
SYSTEMS WITH STABLE POLES GIVE ZERO ENCIRCLEMENTS OF THE ORIGIN AS SEEN BY NEXT TWO EXAMPLES. $G=1/(s+0.2)$ AND $G=10/(s+2)(s+4)$

One can use logic that if $(s+a)$ gives no encirclements of origin (this is D-contour shifted to right by a), then clearly $1/(s+a)$ cannot give encirclements as they have opposite arguments.



SYSTEMS WITH UNSTABLE POLES GIVE ANTI-CLOCKWISE ENCIRCLEMENTS OF THE ORIGIN AS SEEN BY NEXT TWO EXAMPLES. $G=1/(s-0.2)$ AND $G=10/(s-4)(s+2)$

One can use logic that if $(s-a)$ gives clockwise encirclement of the origin (this is D-contour shifted to LEFT by a), then clearly $1/(s-a)$ gives anti-encirclements as they have opposite arguments.



SUMMARY: Allowing s to follow D-contour, it is clear that when plotting the Nyquist diagram:

1. LHP poles and zeros result in no net change in angle and no encirclements (of the origin).
2. RHP zeros give rise to clockwise encirclements of the origin.
3. RHP poles give rise to anti-clockwise encirclements of the origin.

RULES OF COMPLEX NUMBERS

When multiplying complex numbers, the phases add. Therefore the total number of encirclements N of the origin (equivalent to the total change in phase as s describes the D-contour) is given by (Note this is an interim result to be extended next):

$N = \text{number of RHP zeros minus the number of RHP poles}$