Modelling and control summaries

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Nyquist 12: Stability criteria

The full definition of a Nyquist diagram is the mapping of G(s) while s describes the D-contour. We can use this plot to infer the number of closed-loop RHP poles, that is closed-loop stability.

Apply encirclement result to the closed-loop (from previous note).

For the Nyquist diagram of G(s):	**However, really we are interested in closed-loop stability rather than open-loop.**
Number of encirclements of the origin = number of RHP zeros minus the number of RHP poles	

$$G_c = \frac{GK}{1+GK}; \quad \{p_c = 0\} \Rightarrow \{GK = -1\}$$

$$GK = \frac{n}{p_o} \quad \Rightarrow \quad \left\{1 + GK = \frac{n + p_o}{p_o} = \frac{p_c}{p_o}\right\}$$

Closed-loop poles

Open-loop poles

The Nyquist diagram of (1+GK) gives nq=nc-no clockwise encirclements of the origin.
nc = number of RHP closed-loop poles
no = number of RHP open-loop poles.

The Nyquist diagram of GK must give $n_q=n_c-n_o$ clockwise encirclements of -1 point. THIS IS A FACT hence this equation is always satisfied.

How do we infer closed-loop stability?

A system is closed-loop stable if and only if $n_c=0$ (no closed-loop RHP poles).
Therefore, a system is closed-loop stable if and only if $n_q=-n_o$ (implies $n_c=0$).

A system is closed-loop stable if and only if the number of counter-clockwise encirclements of -1 point matches number of open-loop RHP poles.

REMARKS The result is based on two things which students must be skilled at:

1. Counting open-loop RHP poles – a common mistake is to forget the distinction between LHP and RHP poles. Only count open-loop RHP poles. This is n_o.
2. Sketch the Nyquist diagram of the loop transfer function and count encirclements of the -1 point. This is n_q.

Determine the number of RHP closed-loop poles from $n_c=n_q+n_o$.