Modelling and control summaries

by Anthony Rossiter

Nyquist 13: Applying stability criteria

The full definition of a Nyquist diagram is the mapping of G(s) while s describes the D-contour. **A system is closed-loop stable (n_c=number RHP closed-loop poles) if and only if the number of counter-clockwise encirclements of -1 point matches number n_o of open-loop RHP poles.**

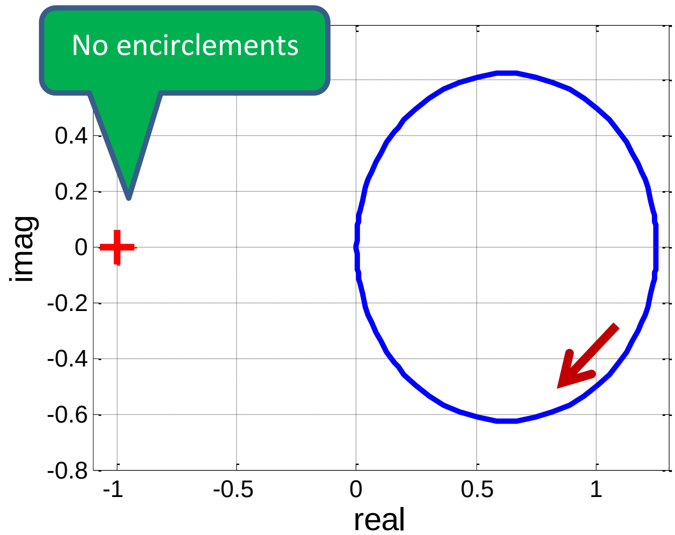
Alternatively, apply $n_q=n_c-n_o$ directly or $n_c=n_q+n_o$

Apply Nyquist stability criteria to numerous examples

$$G=\frac{5}{s+4};$$

$$\left.\begin{array}{l} n_o=0 \\ n_q=0 \end{array}\right\} \Rightarrow \quad n_c=0$$

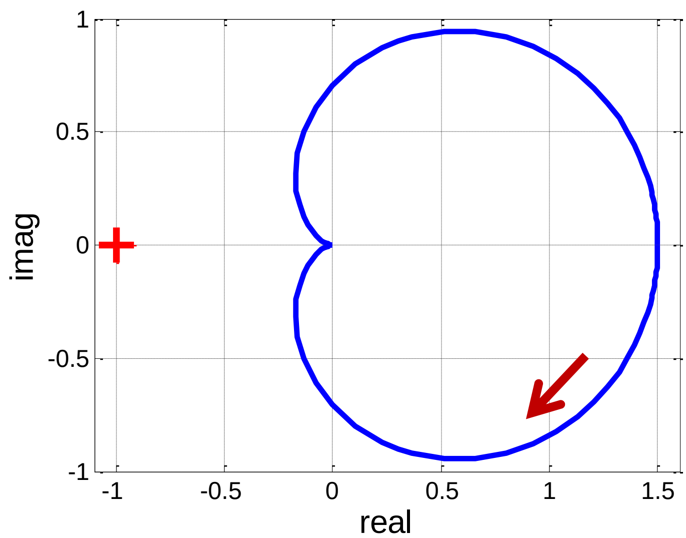
CLOSED-LOOP STABLE (For all +ve K)



$$G=\frac{3}{(s+2)(s+1)};$$

$$\left.\begin{array}{l} n_o=0 \\ n_q=0 \end{array}\right\} \Rightarrow \quad n_c=0$$

CLOSED-LOOP STABLE (For all +ve K)

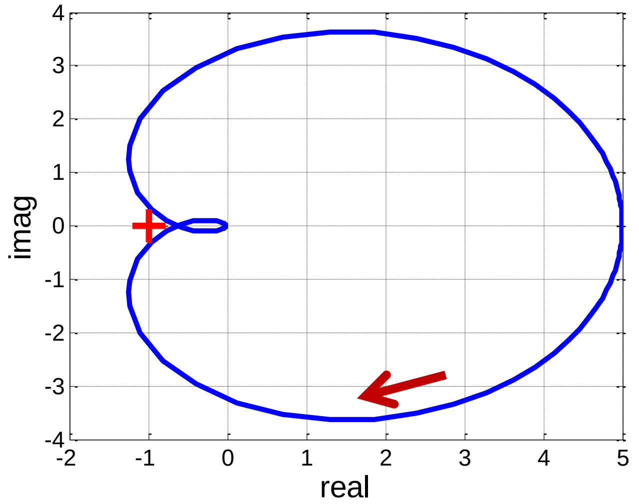


$$G=\frac{5}{(s+1)^3};\quad K=1$$

$$\left.\begin{array}{l} n_o=0\\ n_q=0\end{array}\right\}\Rightarrow\quad n_c=0$$

Clearly stable for current K.

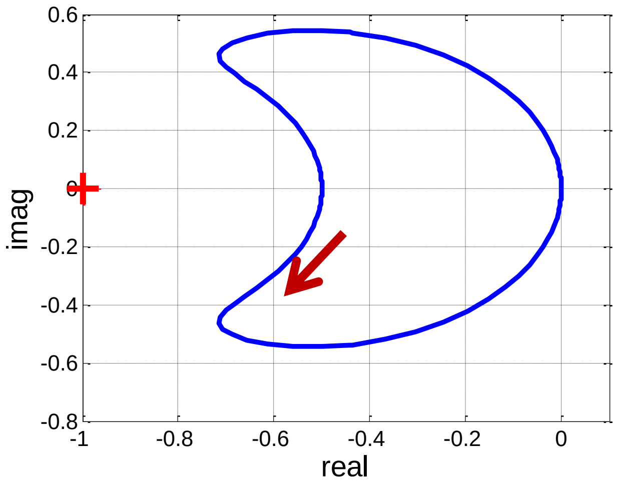
However, K>8/5 implies that n_q=2 and hence one will have n_c=2, that is closed-loop instability.



$$G=\frac{s+0.1}{(s+0.2)(s-1)};\quad K=1$$

$$\left.\begin{array}{l} n_o=1\\ n_q=0\end{array}\right\}\Rightarrow\quad n_c=1$$

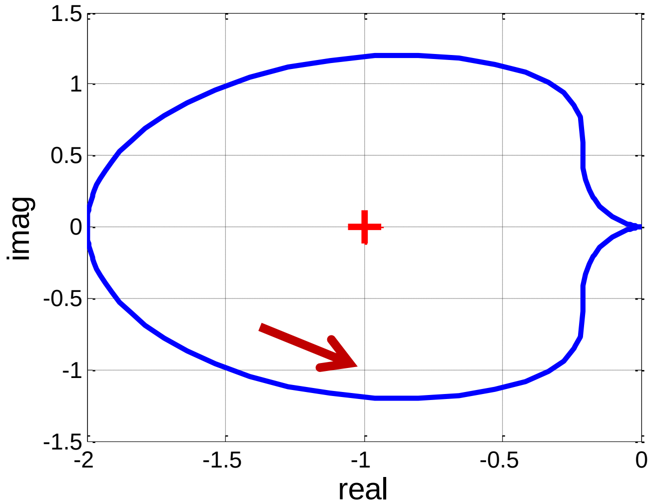
Clearly <u>unstable</u> for current K (a closed-loop RHP pole).
However, K>2 implies that n_q=-1 and hence one will have n_c=0, that is closed-loop stability.



$$G=\frac{40(s+2)}{(s+10)(s+4)(s-1)};$$

$$\left.\begin{array}{l} n_o=1\\ n_q=-1\end{array}\right\}\Rightarrow\quad n_c=0$$

Clearly stable for current K=1.
However, K<0.5 will remove the counter clockwise encirclement and hence closed-loop instability would follow.



<u>REMARKS</u> The result is based on two things which students must be skilled at:

1. Counting open-loop RHP poles – a common mistake is to forget the distinction between LHP and RHP poles. Only count open-loop RHP poles. This is n_o.
2. Sketch the Nyquist diagram of the loop transfer function and count encirclements of the -1 point. This is n_q.

Determine the number of RHP closed-loop poles from $n_c=n_q+n_o$.