Modelling and control summaries

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Nyquist 14: stability criteria & integrators

Apply $n_q=n_c-n_o$ directly or $n_c=n_q+n_o$ (see notes 12,13 for details)

With an integrator, as w ➔0, |G(jw)|➔infinity!

1. D-contour has 2 right hand right angle turns so Nyquist has 2 right hand right angle turns.
2. s (that is D-contour) moves through 180 deg anti-clockwise so Nyquist (that is 1/s) must move through 180 deg clockwise (with infinite gain).

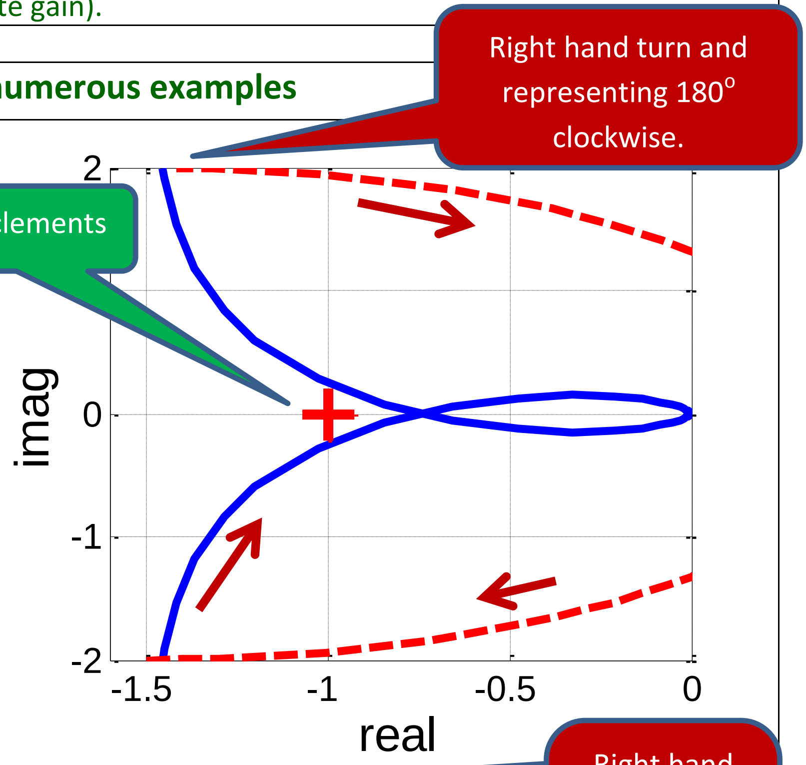
Apply Nyquist stability criteria to numerous examples

$$G=\frac{3}{s(s^2+2s+2)};$$

$$\left.\begin{array}{l} n_o=0 \\ n_q=0 \end{array}\right\} \Rightarrow n_c=0$$

CLOSED-LOOP STABLE

However, if K greater than about 4/3 then 2 encirclements and closed-loop instability results.

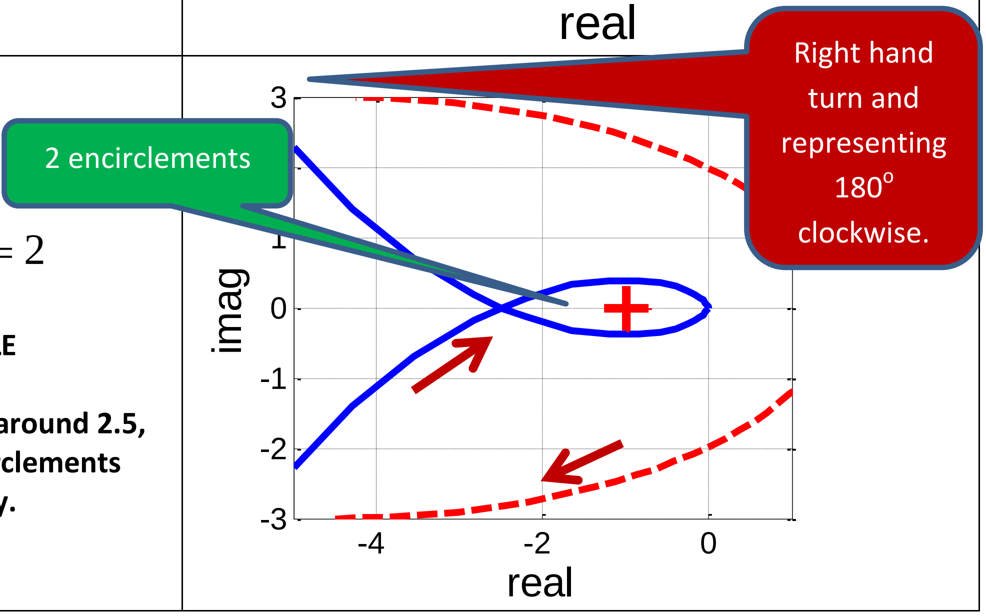


$$G=\frac{5}{s(s+1)^2};$$

$$\left.\begin{array}{l} n_o=0 \\ n_q=2 \end{array}\right\} \Rightarrow n_c=2$$

CLOSED-LOOP UNSTABLE

However, if K less than around 2.5, there would be no encirclements and closed-loop stability.

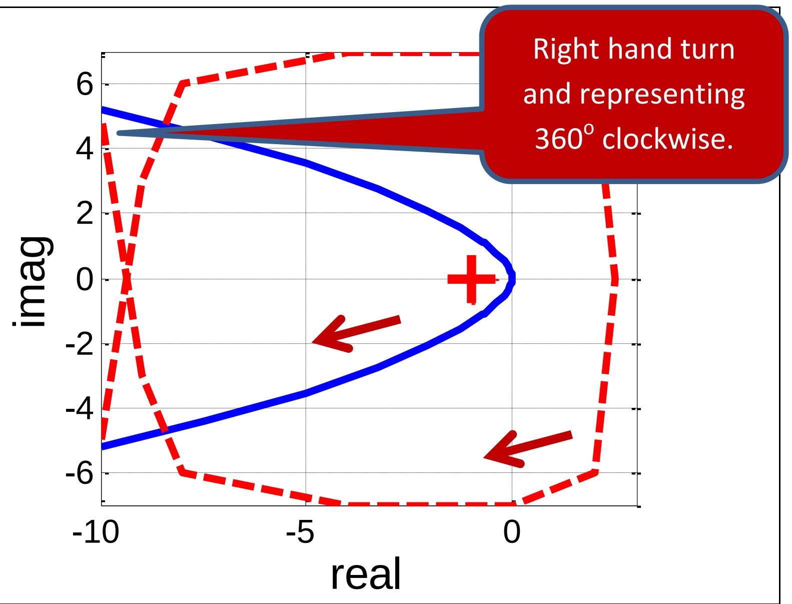


$$G = \frac{s+4}{s^2(s+1)(s+2)};$$

$$\left.\begin{aligned} n_o &= 0 \\ n_q &= 2 \end{aligned}\right\} \Rightarrow n_c = 2$$

Clearly unstable for current K.

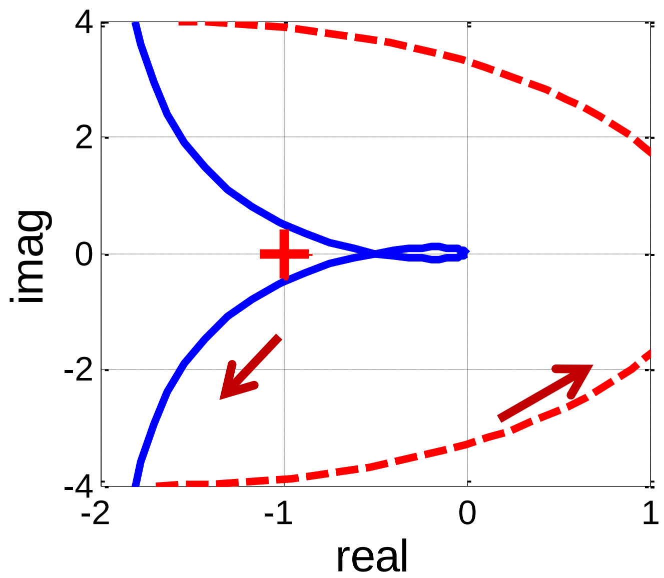
Moreover, unstable for all positive K as changing K does not change the number of encirclements of -1.



$$G = \frac{0.5(s+3)}{s(s-1)}; \quad K = 1$$

$$\left.\begin{aligned} n_o &= 1 \\ n_q &= 0 \end{aligned}\right\} \Rightarrow n_c = 1$$

Clearly unstable for current K (a closed-loop RHP pole).
However, K>2 implies that n_q=-1 and hence one will have n_c=0, that is closed-loop stability.



REMARKS The result is based on two things which students must be skilled at:

1. Counting open-loop RHP poles – a common mistake is to forget the distinction between LHP and RHP poles. Only count open-loop RHP poles. This is n_o.
2. Sketch the Nyquist diagram of the loop transfer function and count encirclements of the -1 point. This is n_q.

Determine the number of RHP closed-loop poles from n_c=n_q+n_o.

- Shown, by example, that it is important to place right hand right angle turns correctly, and also the 180 degree clockwise rotation at infinity, for each integrator.
- Students are reminded to use other tools such as root-loci to check or reinforce their findings.