**Modelling and control summaries**

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**Nyquist 2: Using gain and phase**

<table>
<thead>
<tr>
<th>$u(t)$</th>
<th>$G(s)$</th>
<th>$y(t)$</th>
<th>SUMMARY of Frequency response $G(jw)$</th>
</tr>
</thead>
</table>

### UNDERSTANDING COMPLEX NUMBERS

Students need a clear view on how the gain and phase of a complex number link to a position in the Argand diagram.

**GAIN** is distance from the origin.

**MARK** some examples on an Argand diagram

- $2 \angle 120^\circ$
- $1 \angle -170^\circ$
- $0.5 \angle -45^\circ$
- $2 \angle 30^\circ$

### REMARKS:

1. On easy way to sketch a Nyquist diagram is to transcribe gain and phase information.
2. Students may realise that gain/phase is already available in the Bode diagram! From hereon, assume the Bode diagram is already known.
GUIDELINES FOR QUICK SKETCHES (beginning from Bode)

1. Phase is reducing (or becoming more negative) – plot is moving clockwise.
2. Phase is increasing – plot is moving anti-clockwise.
3. Gain is reducing – plot is moving towards origin.
4. Gain is increasing – plot is moving away from the origin.

You can use arrows to represent this insight.

\[ G = \frac{6}{(s + 1)(s + 3)} \]

\[ |G(jw)| = \frac{6}{\sqrt{(w^2 + 1)(w^2 + 9)}}; \]

\[ \angle G(jw) = -\tan^{-1} w - \tan^{-1} \frac{w}{3} \]

INDICATING TRENDS

From any given point, one can deduce the direction of change of a Nyquist diagram and this enables a quick sketch. Mark arrows for change of phase and change of gain and movement is between these two.

As \( w \) increases,
- plot goes closer to origin (gain decreases)
- plot goes clockwise (phase decreases)