Modelling and control summaries

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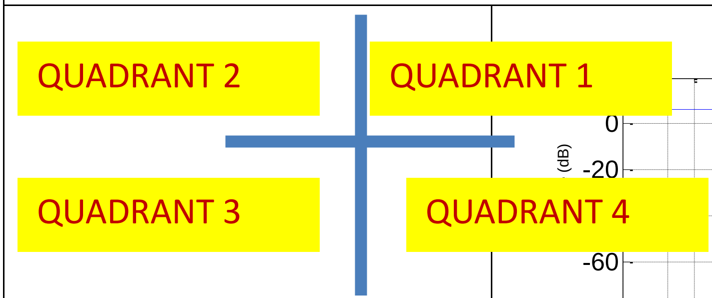
Nyquist 5: Initial quadrant

Asymptotic methods give a rough Bode plot using simple trends: i) as w tends to zero; ii) as w tends to infinity; iii) how do gain and phase change in between.	A simple Nyquist sketch is acquired by transcribing this information into an Argand diagram. NOTE: Accuracy is important near to -1 (0dB and -180 degrees).
A key point is the initial quadrant as this has a big impact on stability inferences covered later.	Where not obvious, students should exercise care in determining the initial quadrant (see nyquist 3,4 for example of how changing zero changed this).

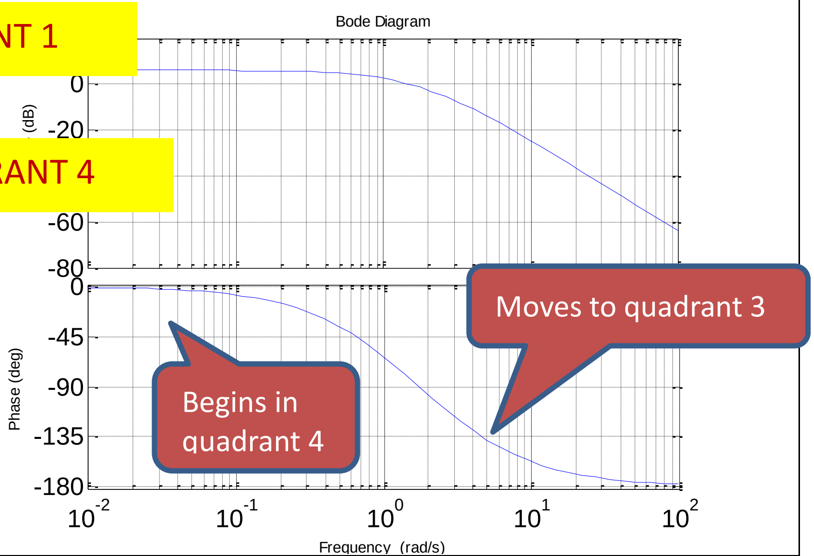
EXAMPLES OF INITIAL QUADRANTS



Quadrant 1: 0 to 90

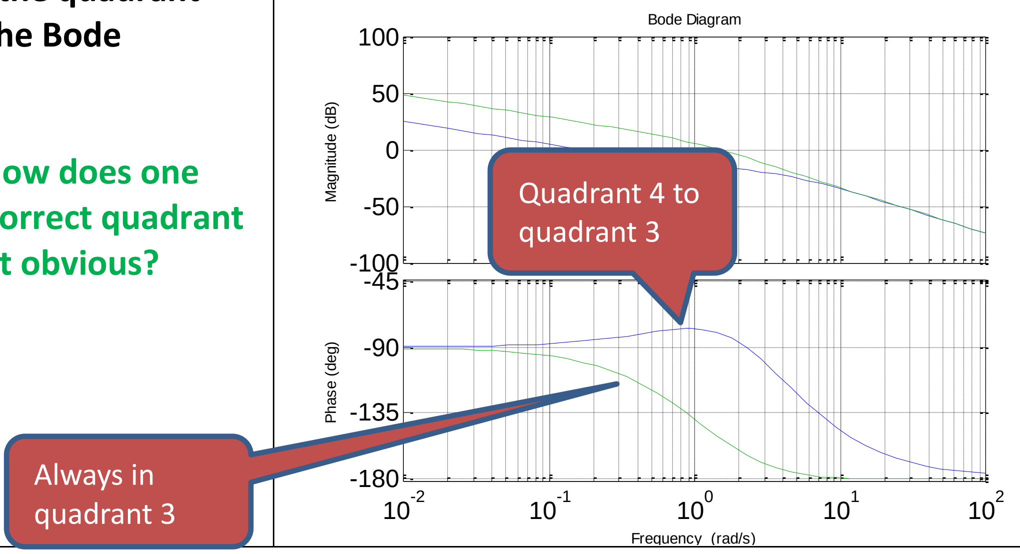
Quadrant 2: 90 to 180
or -180 to -270

Quadrant3: -90 to -180
Quadrant 4: 0 to -90



One can see the quadrant easily from the Bode diagram.

HOWEVER: how does one deduce the correct quadrant when it is not obvious?



LOW FRQUENCY ANALYSIS

Find the phase for a small value of w, this will give the initial quadrant. $w \succ w^2 \succ w^3 \cdots$ Use approximation of poles.	$\frac{1}{jw+a} = \frac{1/a}{1+jw/a} \approx \frac{1}{a}(1-\frac{jw}{a})$

$$G = \frac{9jw+3}{(jw+1)(jw+2)}$$

1. $\frac{1}{jw+1} \Rightarrow (1-jw); \quad \frac{1}{jw+2} \Rightarrow (1-\frac{jw}{2})\frac{1}{2}$ — For small w, approximate poles by a simple zero.

2. $\frac{9jw+3}{(jw+1)(jw+2)} \quad \Rightarrow \quad (9jw+3)(1-jw)(1-\frac{jw}{2})\frac{1}{2}$ — Compute approx. G(jw)

3. $\Rightarrow \quad [3+jw(9-3-\frac{3}{2})+\cdots]\frac{1}{2}$ — Expand and remove high powers of w

$\Rightarrow \quad 3+jw\frac{9}{2} \quad (i.e.\ phase > 0)$ — Indication of G(jw) for small w. Clearly in quadrant 1!

ALTERNATIVE PROCEDURE

Write out the phase explicitly and compute for a small value of w.

$$\frac{9(s+1/3)}{s(s+1)(s+2)}; \quad \frac{5(s+0.6)}{s(s+1)(s+2)}; \quad \frac{3.75(s+0.8)}{s(s+1)(s+2)}; \quad \frac{2(s+3/2)}{s(s+1)(s+2)}$$

$\tan^{-1}\frac{w}{0.33} - \tan^{-1}w - \tan^{-1}\frac{w}{2}; \quad \angle G(j0.1) \approx (8-90)^o$ — Quadrant 4

$\tan^{-1}\frac{w}{0.6} - \tan^{-1}w - \tan^{-1}\frac{w}{2}; \quad \angle G(j0.1) \approx (0.88-90)^o$ — Quadrant 4

$\tan^{-1}\frac{w}{0.8} - \tan^{-1}w - \tan^{-1}\frac{w}{2}; \quad \angle G(j0.1) \approx (-1.44-90)^o$ — Quadrant 3

$\tan^{-1}\frac{2w}{3} - \tan^{-1}w - \tan^{-1}\frac{w}{2}; \quad \angle G(j0.1) \approx (-4.75-90)^o$ — Quadrant 3

Using MATLAB to determine an exact value for phase at small frequency

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>>G=tf(5*[1 0.6],[1 3 2])
>>[g,p]=bode(G,0.01)
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