

Modelling and control summaries



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Nyquist 6: RHP factors and delays

The inclusion of RHP factors means students should be careful with phase computations but otherwise sketching Nyquist uses the same concepts as for LHP factors.
Mark start and end points and use trends in gain and phase inbetween.

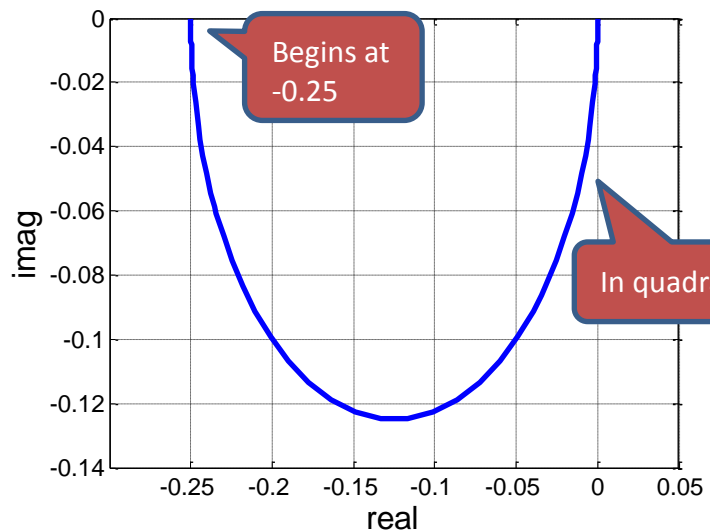
EXAMPLE 1

$$G = \frac{1}{s - 4}$$

$w=0$ gain=-1/4 phase=-180

Gain reduces monotonically as frequency increases.

Phase is $-180 + \tan^{-1}(w/4)$ – this is monotonically increasing from -180 to -90

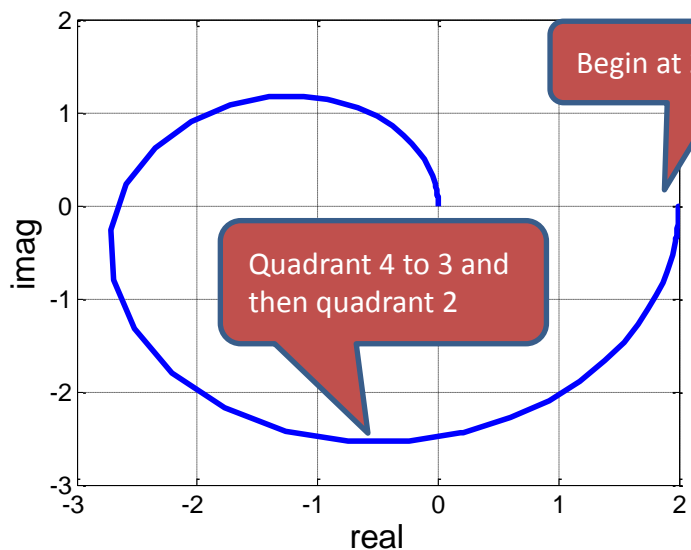


$$G = \frac{16(1-s)}{(s+4)(s+2)}$$

$w=0$ gain=2 phase=0

Gain increases slightly until around $w=1$ and then reduces monotonically as frequency increases.

Phase is given as $-\tan^{-1}(w) - \tan^{-1}(w/2) - \tan^{-1}(w/4)$ so monotonically decreasing from 0 to -270



USE MATLAB TO CHECK YOUR ANSWERS!

SYSTEMS WITH DELAYS

The inclusion of delays makes the sketching of Nyquist a non-paper and pen exercise. You are strongly advised to use a computer. You should however be clear on the impact.

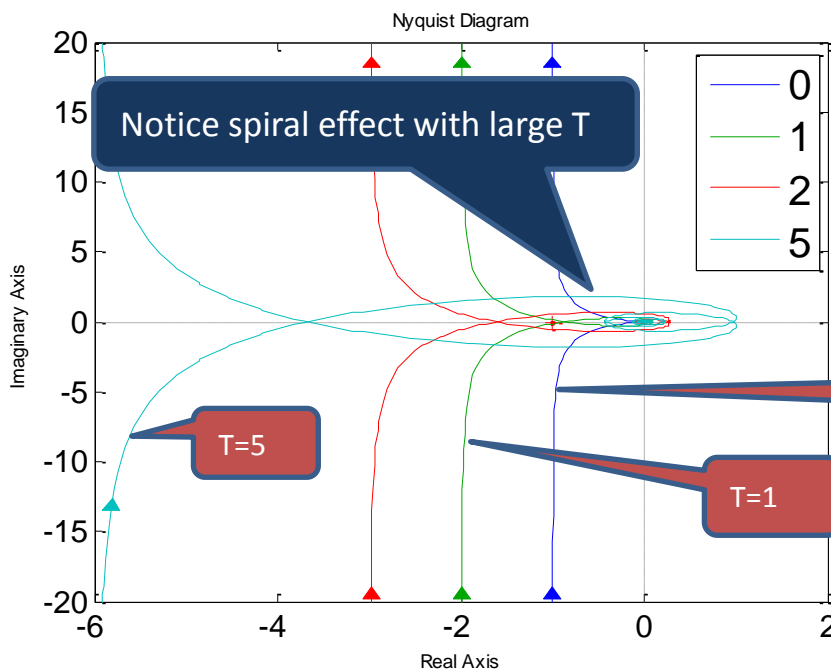
$$G = e^{-sT} H(s), \quad \text{e.g.} \quad e^{-sT} \left(\frac{4(s+2)}{s(s+1)} \right)$$

The delay adds extra phase into the frequency response and thus rotates the Nyquist diagram a different amount at different frequencies.

$$|e^{-j\omega T}| = 1; \quad \angle e^{-j\omega T} = -j\omega T$$

$$\angle G = \angle H - \omega T$$

This frequency dependent phase shift causes a spiral effect in the Nyquist diagram for large frequencies as every time ωT increases by 2π , a full extra rotation takes place.



$$G = e^{-sT} \left(\frac{1}{s(s+1)} \right)$$

ENTERING DELAYS ON MATLAB

```
G=tf(1,[1 1 0], 'iodelay', 5);
bode(G)
nyquist(G)
```