The full definition of a Nyquist diagram is the mapping of $G(s)$ while $s$ describes the D-contour. Therefore we need to describe the D-contour fully in order to form a complete Nyquist diagram.

<table>
<thead>
<tr>
<th>D-contour</th>
<th>Nyquist Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. D-contour comprises the full imaginary axis (skirting around the origin) and then encircles the RHP. Move in direction of increasing frequency.</td>
<td>Right angle turn</td>
</tr>
<tr>
<td>2. Special care must be given to include 4 right hand right angle turns: 2 to skirt origin and 2 to encircle RHP.</td>
<td>Two right angle turns to avoid origin</td>
</tr>
<tr>
<td>3. The D-contour comprises all the values of $s$ used to sketch the Bode and Nyquist plots.</td>
<td>Right angle turn</td>
</tr>
<tr>
<td>4. Nyquist diagram is a mapping of $G(s)$ as $s$ describes the D-contour.</td>
<td></td>
</tr>
<tr>
<td>5. Nyquist must contain a RH right angle turn at points corresponding to those in the D-contour.</td>
<td></td>
</tr>
</tbody>
</table>

\[ G(s) = \frac{N(s)}{H(s)} \]

\[ G = \text{tf}(4, [1 5 0]); \]
\[ H = \text{tf}(10 20), [1 10 25]) \]
\[ M = \text{tf}(2 20), \text{poly}(0 -1 -4)) \]
\[ P = \text{tf}(3, \text{poly}(1 -6)) \]
\[ N = \text{tf}(0.0005*1 0.02), \text{poly}(0.008,-0.06)*(-1)) \]
\[ L = \text{tf}(1 2), [1 4 0 0]) \]

**NYQUIST DIAGRAM IS SYMMETRICAL ABOUT REAL AXIS BECAUSE INCLUDES BOTH NEGATIVE AND POSITIVE FREQUENCIES AND:**

\[ G(jw) = \text{conj}(G(-jw)) \]
### CONFORMAL MAPPINGS

For an analytic function \( G(s) \), if locus of \( s \) moves through an angle ‘a’, then locus of \( G(s) \) must also move through an angle ‘a’.

The D-contour includes 4 right hand right angle turns. Two of these are important, the ones around the origin. The corresponding points in the Nyquist diagram must also include right hand right angle turns.

**NOTE:** Emphasis here is on two properties

1. **RIGHT ANGLE**
2. **RIGHT HAND**

In practice conformal mappings are only needed to deal with \( w \rightarrow 0 \) for systems with integrators (thus gain \( \rightarrow \infty \)). This means that at those points the Nyquist diagrams has a right hand right angle turn and this directionality can be used to compute encirclements.

With no integrators, the right hand turns are not noticed as ‘s=0’ is dominated by ‘a’ in \( (s+a) \).

### NYQUIST FOR AN INTEGRATOR

The D-contour describes the loci for \( s \). An integrator is given as \( 1/s \).

1. Therefore the argument is given as the opposite argument to the D-contour (from properties of complex numbers).
2. The D-contour is anti-clockwise around the origin for small \( w \), so the mapping through \( (1/s) \) must be clockwise with infinite gain).
3. Also has right hand right angle turns.

### Further insights linked to conformal mapping

Closed-loop stability seemed to be linked to whether the Nyquist diagram passed with the -1 point on the right or the left. (Direction is increasing \( w \).)

This corresponds to whether (as one moves along the D-contour) one is looking to the right (into the RHP for a solution) or the left (into the LHP for a solution).

- **Passing with -1 point to the right** indicates a likelihood of having a closed-loop pole in the RHP. A simple indicator of closed-loop stability is that one passes with -1 to the left.

- **Look to right**
- **Look to left**