Modelling and control summaries

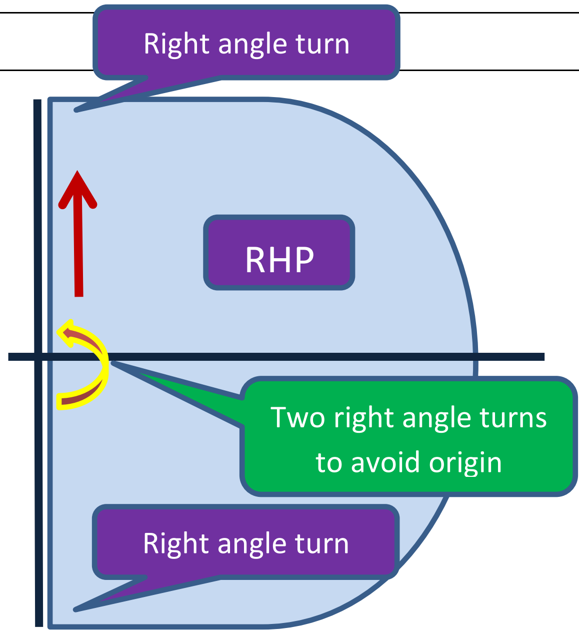
by Anthony Rossiter

Nyquist 9: D-contour

The full definition of a Nyquist diagram is the mapping of G(s) while s describes the D-contour. Therefore we need to describe the D-contour fully in order to form a complete Nyquist diagram.

D-contour

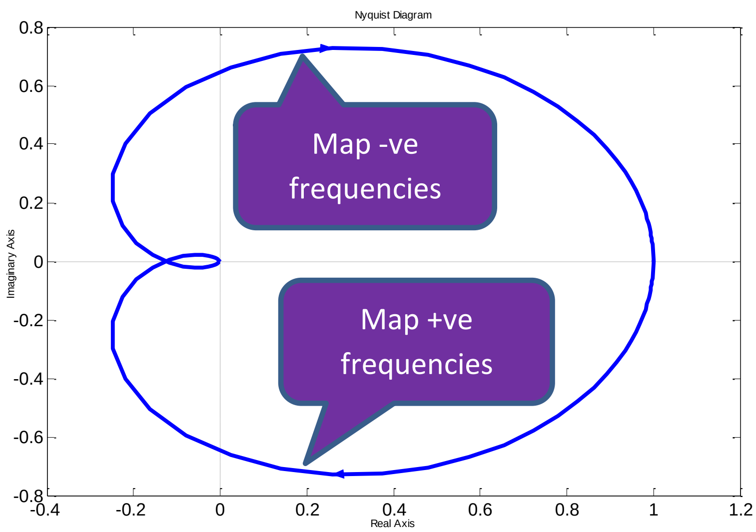
1. D-contour comprises the full imaginary axis (skirting around the origin) and then encircles the RHP. Move in direction of increasing frequency.
2. Special care must be given to include 4 right hand right angle turns: 2 to skirt origin and 2 to encircle RHP.
3. The D-contour comprises all the values of s used to sketch the Bode and Nyquist plots.
4. Nyquist diagram is a mapping of G(s) as s describes the D-contour.
5. Nyquist must contain a RH right angle turn at points corresponding to those in the D-contour.



G=tf(4,[1 5 0]);	**P=tf(3,poly([1 -6]))**
H=tf([10 20],[1 10 25])	**N=tf(0.0005*[1 0.02],poly([0.008,-0.06])*(-1))**
M=tf([2 20],poly([0 -1 -4]))	**L=tf([1 2],[1 4 0 0])**

NYQUIST DIAGRAM IS SYMETRICAL ABOUT REAL AXIS BECAUSE INCLUDES BOTH NEGATIVE AND POSITIVE FREQUENCIES AND:

$$G(jw) = conj(G(-jw))$$



CONFORMAL MAPPINGS

For an analytic function G(s), if locus of s moves through an angle 'a', then locus of G(s) must also move through an angle 'a'.

The D-contour includes 4 right hand right angle turns. Two of these are important, the ones around the origin. The corresponding points in the Nyquist diagram must also include right hand right angle turns.

NOTE: Emphasis here is on two properties

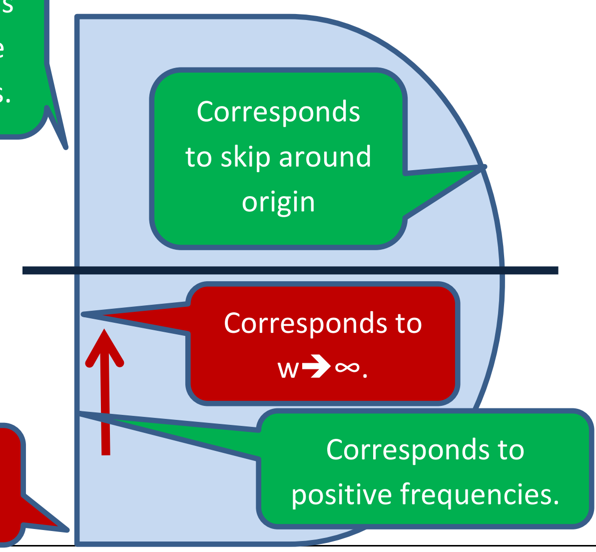
1. RIGHT ANGLE
2. RIGHT HAND

In practice conformal mappings are only needed to deal with w➔0 for systems with integrators (thus gain ➔∞). This means that at those points the Nyquist diagrams has a right hand right angle turn and this directionality can be used to compute encirclements.
With no integrators, the right hand turns are not noticed as 's=0' is dominated by 'a' in (s+a).

NYQUIST FOR AN INTEGRATOR

The D-contour describes the loci for s.
An integrator is given as 1/s.

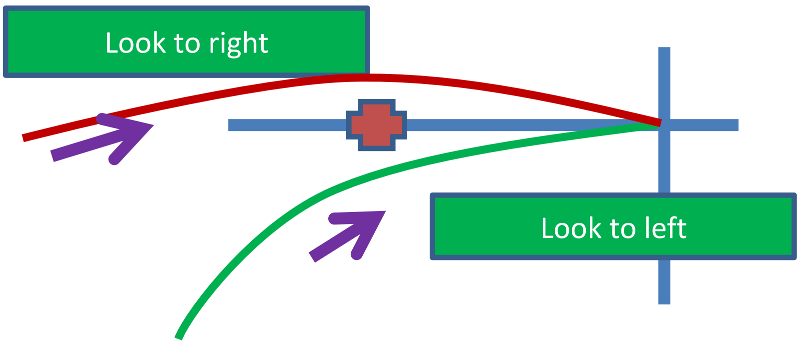
1. Therefore the argument is given as the opposite argument to the D-contour (from properties of complex numbers).
2. The D-contour is anti-clockwise around the origin for small w, so the mapping through (1/s) must be clockwise with infinite gain).
3. Also has right hand right angle turns.



Further insights linked to conformal mapping

Closed-loop stability seemed to be linked to whether the Nyquist diagram passed with the -1 point on the right or the left. (Direction is increasing w.)

This corresponds to whether (as one moves along the D-contour) one is looking to the right (into the RHP for a solution) or the left (into the LHP for a solution).



Passing with -1 point to the right indicates a likelihood of having a closed-loop pole in the RHP. A simple indicator of closed-loop stability is that one passes with -1 to the left.