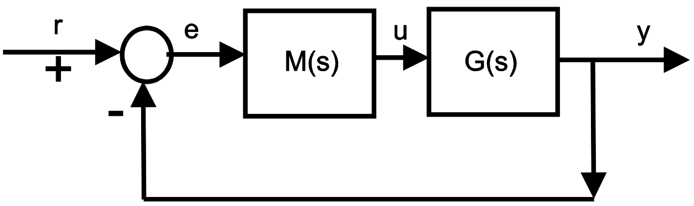
Modelling and control summaries

by Anthony Rossiter

Root-loci 11: Using a sketching for design

The focus is on the simplest form of block diagram, a process *G(s)* and a compensator *M(s)* which is expressed as a gain K multiplied by a transfer function. The closed-loop transfer function is $G_c(s)$.



$$M(s) = K\tilde{M}(s)$$

$$G_c = \frac{GK\tilde{M}}{1+GK\tilde{M}} = \frac{Kn}{Kn+d}$$

Conditions for closed-loop poles

$$p_c = 0 \quad \equiv \quad \left\{ \begin{array}{c} Kn + d = 0 \\ K\frac{n}{d} = -1 \\ \arg\left(\frac{n}{d}\right) = \pm 180^o \end{array} \right\}$$

Summary of rules for design

1. *A key aim is to place the closed-loop poles in desirable positions.*
2. *A root-loci plot gives a pictorial view of the possible pole positions, assuming only gain changes.*
3. *Hence, one need only identify where you want to be on the loci, and then find the K which puts you there.*

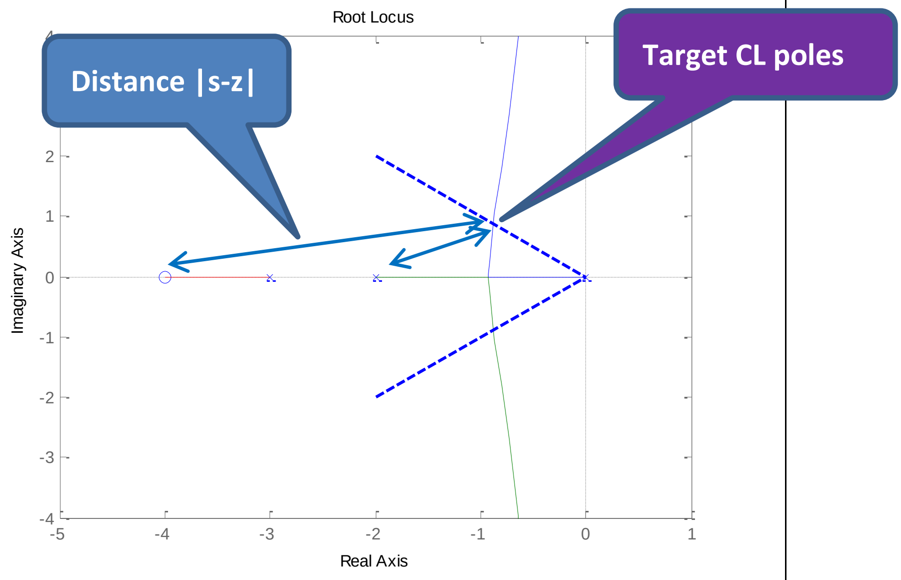
This process is easy on MATLAB (say with sisotool), but one can do good quality estimates by hand using a minimum of computation, as illustrated here.

For simplicity, this note will assume that the desired dominant pole locations have a damping ratio of about 0.71, that is the real part matches the imaginary part.

Key closed-loop pole condition to be deployed:	**|Kn/d|=1**
One can easily rearrange the closed-loop pole condition to find an explicit expression for the require K to give a closed-loop pole at a specified position s.	$K = \frac{\|d(s)\|}{\|n(s)\|} = \frac{\|d_0(s-p_1)\cdots(s-p_k)\|}{\|n_0(s-z_1)\cdots(s-z_m)\|}$
One can solve this equation graphically as each norm is the product of distances to the closed-loop poles/zeros respectively.	$\|d(s)\| = d_0\|s-p_1\|\cdots\|s-p_k\|$ $\|d(s)\| = n_0\|s-z_1\|\cdots\|s-z_k\|$

$$G = \frac{s+4}{s(s+2)(s+3)}$$
$$M(s) = K$$

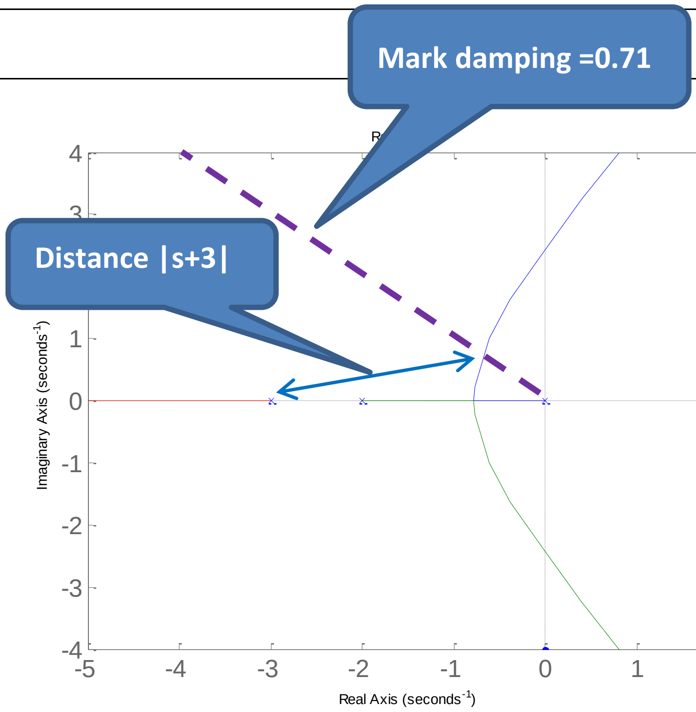
The dotted lines mark where the real part and imaginary part are the same.
This intercepts the loci at approx. s=-0.8+j0.8
The norms of $(s-p_i)$ can be determined by Pythagorous, or visually (see double sided arrows).



$$K = \frac{|s(s+2)(s+3)|}{|s+4|} = \frac{0.8\sqrt{2} \times \sqrt{0.64+1.2^2} \times \sqrt{0.64+2.2^2}}{\sqrt{0.64+3.2^2}}$$

$$G = \frac{1}{s(s+2)(s+3)}$$
$$M(s) = K$$

The dotted lines mark where the real part and imaginary part are the same.
This intercepts the loci at approx. s=-0.6+j0.6
The norms of (s-pi) can be determined by Pythagorous, or visually (see double sided arrows).



$$K = \frac{|(s-p_1)||(s-p_2)||(s-p_3)|}{1} = \frac{|0.6(1+j)||1.4+0.6j||2.4+0.6j|}{1}$$
$$K = \sqrt{0.72 \times (1.96+0.36) \times (5.76+0.36)} = 3.2$$