# Modelling and control summaries

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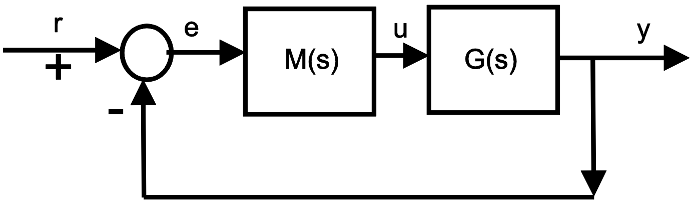
# Root-loci 13: Effects of lag compensation

The focus is on the simplest form of block diagram, a process *G(s)* and a compensator *M(s)* which is expressed as a gain K multiplied by a transfer function. The closed-loop transfer function is $G_c(s)$.




**Lag compensator**

$$M(s) = K\frac{s+ra}{s+a}$$

$$1 \le r \le 10$$

**How does the addition of a lag compensator affect a root-loci plot?**

1. *Adds a real pole and real zero, so no change to asymptote directions.*
2. *Asymptote centroid must change.*
3. *Parts of loci on real axis will change.*

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1. *Sum (OL poles) – sum (OL zeros) = (k-m)*Centroid*
2. *The lag adds a zero at (-ra) and a pole at (-a), therefore* ***the centroid moves to the right (towards RHP)****:*

$$\delta Centroid = \frac{-a+ra}{k-m}; \quad r>1 \quad \Rightarrow \quad \delta Centroid > 0$$

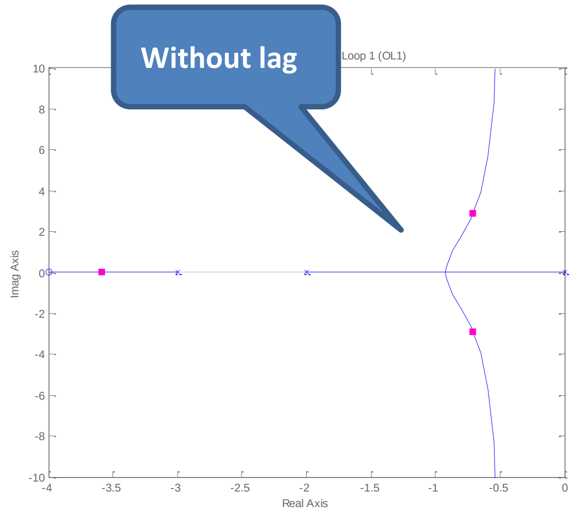
**SUMMARY: As a lag moves the centroid right, one might expect oscillation (under-damping) with a smaller value of gain and hence Lag is a low gain strategy. Also, the real parts of the dominant poles are nearer the RHP, so the loop is slower.**

**One expects *ra* to be small compared to the initial centroid, otherwise the centroid could be moved into the RHP which means one is in danger of getting closed-loop instability.**

**Lag IS not appropriate for open-loop unstable systems which already have a centroid pushed right by the right half plane pole.**

Illustration of a poor choice. The lag has moved the centroid into the RHP and the system is unstable with small values of K.

$$G=\frac{s+4}{s(s+2)(s+3)};\quad K(s)=K\frac{s+5}{s+3};\quad \delta C=\frac{3(\frac{5}{3}-1)}{2}=1$$



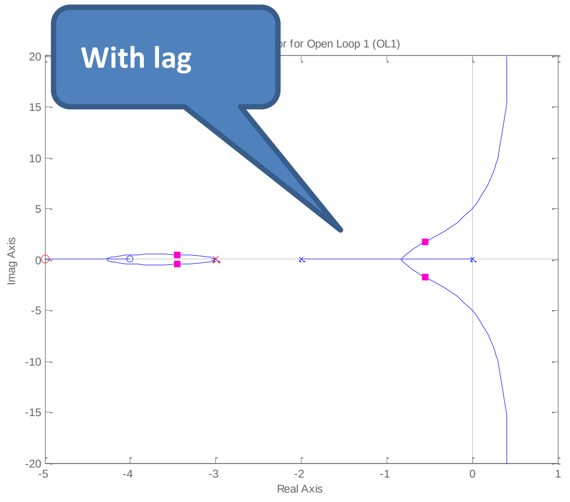
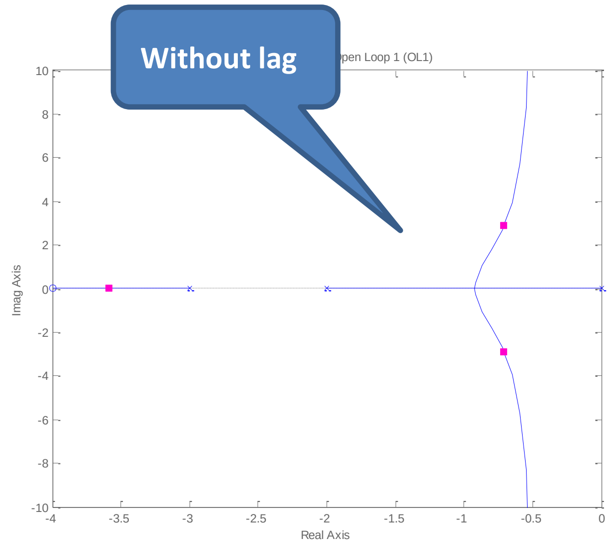
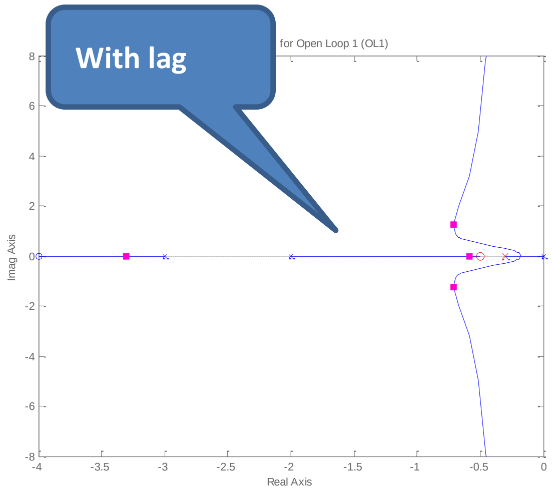




Illustration of a better choice. The lag has moved the centroid only slightly and the system is stable with all values of K.

$$G=\frac{s+4}{s(s+2)(s+3)};\quad K(s)=K\frac{s+0.5}{s+0.3};\quad \delta C=\frac{0.3(\frac{5}{3}-1)}{2}=0.1$$




## TRY THESE EXAMPLES FOR YOURSELF

$$G1=\frac{4}{(s+2)(s+1)};\quad K=0.57;\quad K_{lag}=0.57\frac{s+0.2}{s+0.05}$$

$$G2=\frac{s+4}{s(s+2)(s+3)};\quad K=1.25;\quad K_{lag}=1.25\frac{s+0.2}{s+0.05}$$

$$G=\frac{3}{(s-2)(s+3)};$$

$$K(s)=K\frac{s+5}{s+3}$$