Modelling and control summaries

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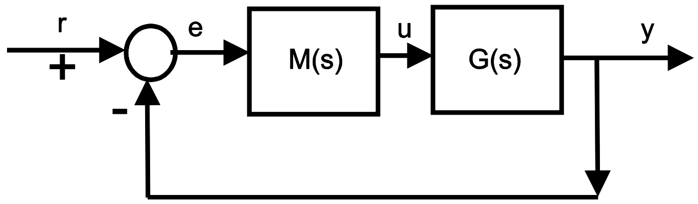
Root-loci 14: Effects of lead compensation

The focus is on the simplest form of block diagram, a process *G(s)* and a compensator *M(s)* which is expressed as a gain K multiplied by a transfer function. The closed-loop transfer function is $G_c(s)$.



Lead compensator

$$M(s) = K\frac{s+a}{s+ra}$$
$$1 \le r \le 10$$

How does the addition of a lead compensator affect a root-loci plot?

1. *Adds a real pole and real zero, so no change to asymptote directions.*
2. *Asymptote centroid must change.*
3. *Parts of loci on real axis will change.*

1. *Sum (OL poles) – sum (OL zeros) = (k-m)*Centroid*
2. *The lead adds a zero at (-a) and a pole at (-ra), therefore* ***the centroid moves to the left (towards LHP)****:*

$$\delta Centroid = \frac{-ra+a}{k-m}; \quad r>1 \quad \Rightarrow \quad \delta Centroid < 0$$

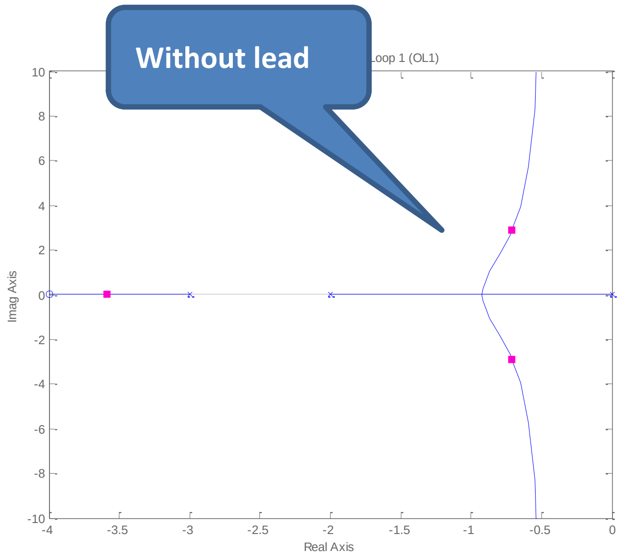
SUMMARY: As a lead moves the centroid left, one might less oscillation (better damping) with the same value of gain and hence Lead is a high gain strategy. Also, the real parts of the dominant poles are further into the LHP, so the loop is faster.

One expects *ra* to be <u>significant</u> compared to the initial centroid, otherwise the centroid would not be moved much and the impact would be small.

Lead IS appropriate for open-loop unstable systems as these have a centroid pushed right by the right half plane pole and thus a movement left is essential. However may get a loss in steady-state gain.

Illustration of impact of lead in pulling asymptotes to the left and also allowing larger values of K.

$$G=\frac{s+4}{s(s+2)(s+3)};\quad K(s)=K\frac{s+3}{s+5};\quad \delta C=\frac{-5+3}{2}=-1$$



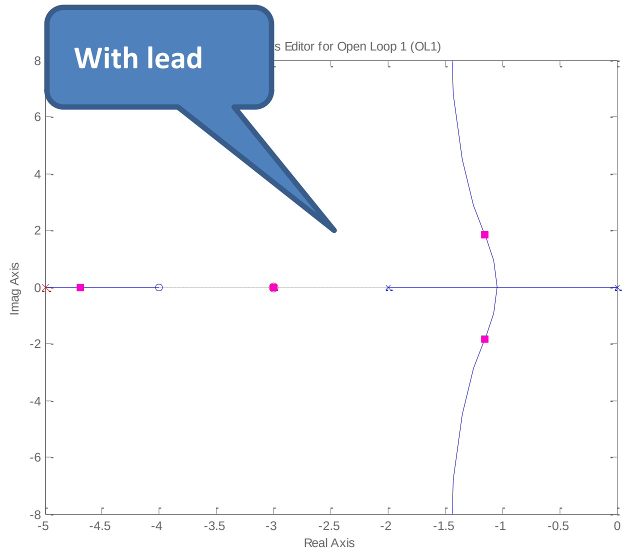
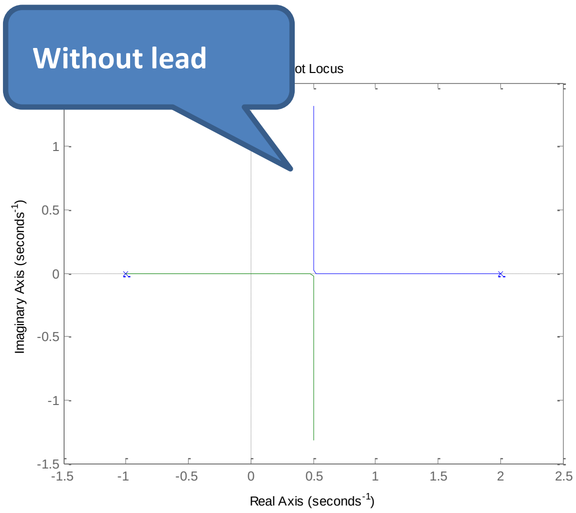
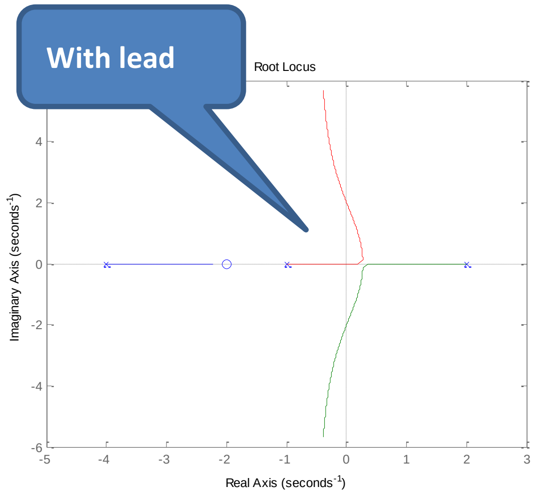


Illustration of a Lead being use to stabilise an open-loop unstable process by moving asymptotes into LHP.

$$G=\frac{1}{(s+1)(s-2)};\quad K(s)=K\frac{s+2}{s+4};\quad \delta C=\frac{-4+2}{2}=-1$$





TRY THESE EXAMPLES FOR YOURSELF Compare and contrast the closed-loop systems with the following compensators [loci, responses and offset].

$$G1=\frac{4}{(s+2)(s+1)};\quad K=0.6;\quad K_{lead}=1.6\frac{s+1}{s+3}$$

$$G2=\frac{s+4}{s(s+2)(s+3)};\ K=1.25;\quad K_{lead}=3.4\frac{s+1}{s+3}$$