Modelling and control summaries

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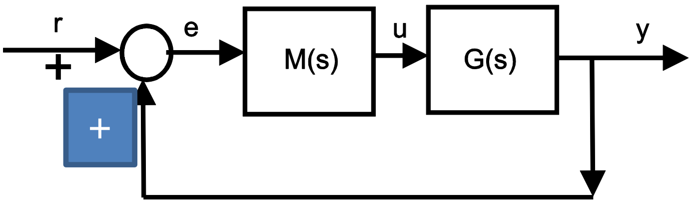
Root-loci 15: Rules with positive feedback

The key closed-loop relationships are subtly different with positive feedback. Assume process *G(s)* and a compensator *M(s)* which is expressed as a gain K multiplied by a transfer function.



$$M(s) = K\tilde{M}(s)$$

$$G_c = \frac{GK\tilde{M}}{1 - GK\tilde{M}} = \frac{Kn}{Kn - d}$$

Conditions for closed-loop poles

$$p_c = 0 \quad \equiv \quad \left\{ \begin{array}{c} Kn - d = 0 \\ K\dfrac{n}{d} = 1 \\ \arg\left(\dfrac{n}{d}\right) = 0^o \end{array} \right\}$$

Summary of rules for sketching and comparison with negative feedback

Rule 1: Mark OL poles with a X

Rule 2: Mark OL zeros with a O

Rule 3: Compute asymptote directions from excess poles over zeros. **[CHANGES]**

Rule 4: Compute asymptote centroid and add asymptotes to plot.

Rule 5: Add parts of loci on real axis. **[CHANGES]**

Note that loci always depart the real-axis at a 90 degree angle

RULE 3: Key Observations about large numbers (see notes rootloci7)

$$\left. \begin{array}{l} s \to \infty \\ K \to \infty \end{array} \right\} \Rightarrow \quad Kn - d \to Kn_m s^m - d_k s^k; \quad Kn_m s^m - d_k s^k = 0 \quad \Rightarrow \quad (Kn_m - d_k s^{k-m}) = 0$$

$$s = \sqrt[k-m]{Kn_m / d_k}$$

SUMMARY: Asymptotic directions are (k-m)th roots of 1.

k-m=1: +ve real axis

k-m=2: +ve and –ve real axis

etc.

$$k - m = 2 \quad \Rightarrow \quad \sqrt{1} = \quad 0^o, 180^0$$

$$k - m = 3 \quad \Rightarrow \quad \sqrt[3]{1} = \quad 120^0, -120^0, 0^0$$

$$k - m = 4 \quad \Rightarrow \quad \sqrt[4]{1} = 0^o, 180^0, \pm 90^o$$

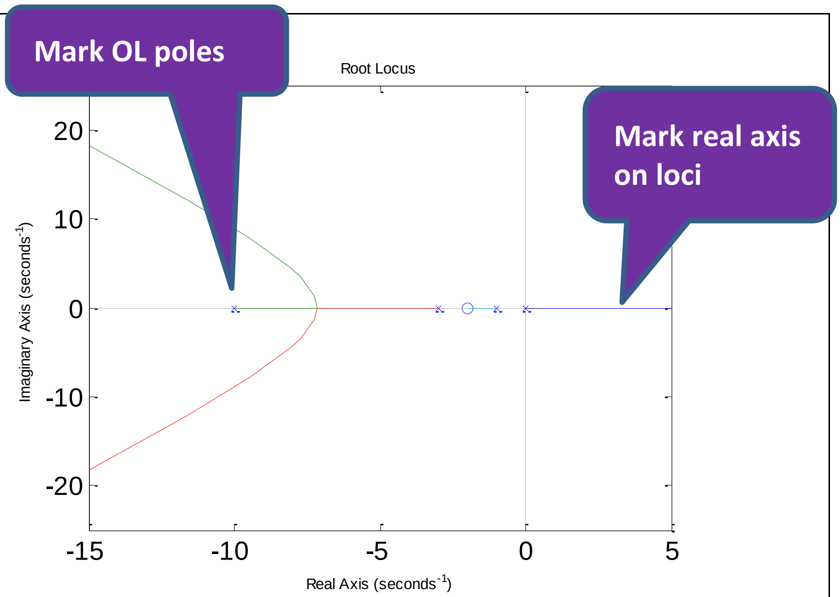
RULE 5: Determine arg(n/d) when s is on the real axis (see notes rootloci8)	
$n=(s-z_1)(s-z_2)\cdots(s-z_m) \quad \Rightarrow \quad \angle n=\sum\angle(s-z_i)$ $d=(s-p_1)(s-p_2)\cdots(s-p_k) \quad \Rightarrow \quad \angle d=\sum\angle(s-p_i)$	
Phase of n(s) is the number of zeros to the right of s multiplied by 180°. **Phase of d(s) is the number of poles to the right of s multiplied by 180°.** $\angle n-\angle d=$ **(Number of poles + zeros to the right)*180°.**	
$\angle n(s)-\angle d(s)=(\pm n360)^o \quad \Rightarrow$	**There exists a K such that Kn/d=1 and hence that value of s is a possible closed-loop pole.**

SUMMARY: If there are a EVEN number of (poles+zeros) to the right, then that part of the real axis must lie on the loci.

Apply rules 1 to 5.

$$G=\frac{s+2}{s(s+10)(s+3)(s+1)}$$
$$M(s)=K$$

3 asymptotes
Centroid =(-14+2)/3=-4
Asymptote directions are 0 and plus/minus 120.
Real axis on loci if 0,2,4 (poles+zeros) to the right.



Root-loci rules were derived assuming all factors were written as monic, that is (s+a) or (s-b). What if a factor is written as (b-s)?

Negative feedback with $G=\frac{3(2-s)}{s(s+1)(s+3)}$ is equivalent to G with monic factors and positive feedback. $G=\frac{3(s-2)}{s(s+1)(s+3)}\times(-1)$	Negative feedback with $G=\frac{0.3(s+0.5)}{s(2+s-s^2)}$ is equivalent to G with monic factors and positive feedback. $G=\frac{0.3(s+0.5)}{s(s-2)(s+1)}\times(-1)$	• Negative feedback rules apply when all factors are monic and the left over scalar multiplier is positive. • Positive feedback rules apply when all factors are monic and the left over scalar multiplier is negative.