Modelling and control summaries

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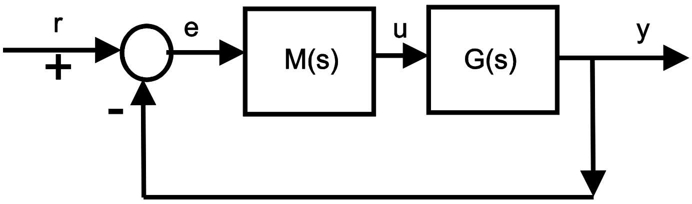
Root-loci 16: Breakaway points

Consider a process *G(s)* and a compensator *M(s)* which is expressed as a gain K multiplied by a transfer function. The closed-loop transfer function is $G_c(s)$.



$$GK\tilde{M} = K\frac{n(s)}{d(s)} \quad \Rightarrow \quad G_c = \frac{Kn}{\underbrace{Kn+d}_{p_c}}$$

When is there a repeated pole?

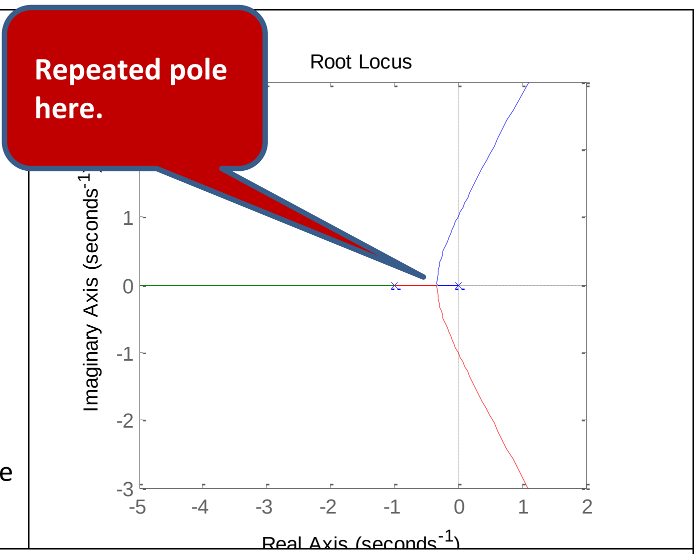
Significance of repeated poles

This is where the root-loci will leave (or re-join) the real-axis and hence knowing their values, or approximate values, is useful for sketching.

It is a rather tedious paper and pen exercise to compute these precisely, so I would recommend using some simple estimation and common sense rather than explicit computation.

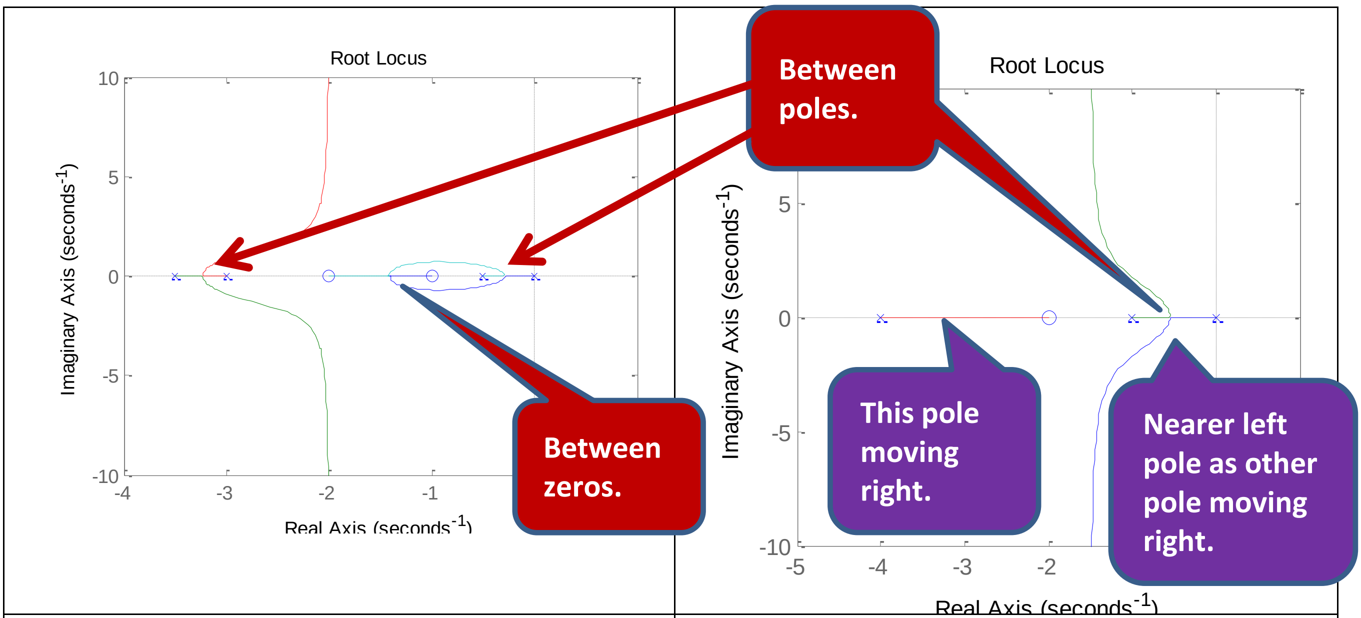
If you need an exact plot, use a computer!

Denoted break away points as this is where loci leave the real-axis.



Obvious observations

- **Breakaway points must occur where the real-axis is on the loci.**
- **Must occur on parts of axis between 2 poles or 2 zeros because either the 2 poles are eventually going to asymptotes or, as zeros are attractors, 2 loci must being joining axis and separating one each for each zero.**
- **Where the number of poles/zeros is small, one can estimate whether the breakaway point is nearer to one of the nearest two poles by considering the expected movement of the other loci remembering that the sum of closed-loop poles is constant, for k-m>1.**



$G=\dfrac{(s+1)(s+2)}{s(s+0.5)(s+3)(s+3.5)}$	$G=\dfrac{s+2}{s(s+1)(s+4)}$

<u>Calculation of breakaway points can be done algebraically by noting this gives a repeated root of the closed-loop pole-polynomial.</u>

$p_c = Kn + d$	Let z be the repeated root. Then z is also a root of the derivative of p_c.

STEP1: Find the derivative of p_c.
STEP 2: Find the roots of the derivative. One of these, hopefully obvious, will be the repeated root.

REMINDER: In general, these computations cannot be done on pen and paper.

EXAMPLE:

$$G=\frac{1}{s(s+1)(s+2)}$$

$$p_c = s^3+3s^2+2s+K$$

There exists K such that s is a closed-loop pole if:

$$\frac{dp_c}{ds}=3s^2+6s+2=0$$

$$s=\frac{-6\pm\sqrt{36-24}}{6}=-1\pm\frac{\sqrt{3}}{3}$$

CLEARLY breakaway point is larger of these.