Modelling and control summaries

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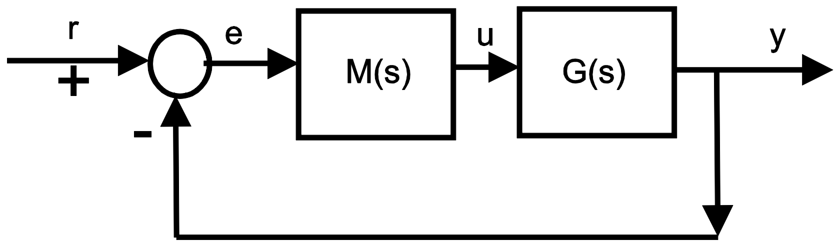
Root-loci 2: Impact of changing poles

This brief summary assumes readers are familiar with the concept of feedback and block diagrams. For simplicity the focus is on the simplest form of block diagram, although it is relatively straightforward to extend the concepts to more complex arrangements. Hence consider a process *G(s)* and a compensator *M(s)* which is expressed as a gain K multiplied by a transfer function. The closed-loop transfer function is $G_c(s)$.



$$M(s) = K\tilde{M}(s)$$

$$G_c = \frac{GK\tilde{M}}{1+GK\tilde{M}}$$

What is the closed-loop pole polynomial dependence on gain? It helps to separate the controller gain from the dynamic parts so the closed-loop pole polynomial can be written down explicitly in terms of this gain.	$GK\tilde{M} = K\frac{n(s)}{d(s)} \Rightarrow G_c = \frac{Kn}{\underbrace{Kn+d}_{p_c}}$
How does closed-loop behaviour change with compensator gain? Because behaviour is closely linked to K, this note motivates the need to understand the links better.	• **As the poles change, so will behaviour.** • **This note illustrates the impact, but without insight.**

Illustrate the closed-loop behaviour dependence upon the choice of compensator gain

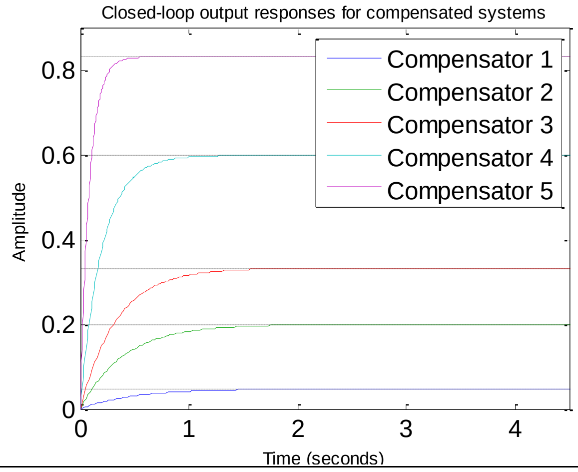
1. It is clear from the examples that changing K has a significant impact on the pole positions and this in turn has a significant impact on the closed-loop behaviour.
2. Examples 1-3 show that the impact is not uniform in that for some cases increasing K helps and for others it may or may not. Often the best value is a form of middle value.
3. We need some analytical tools to understand how changes in K affect behaviour so that we can select this in a systematic fashion.

EXAMPLE 1:

$$\left\{G(s)=\frac{1}{s+2};\quad M(s)=K\right\}$$
$$\Rightarrow\quad p_c=s+2+K$$

$$K=0.1,0.5,1,3,10$$

As gain increases, the response is faster and the steady-state is higher (or offset smaller).

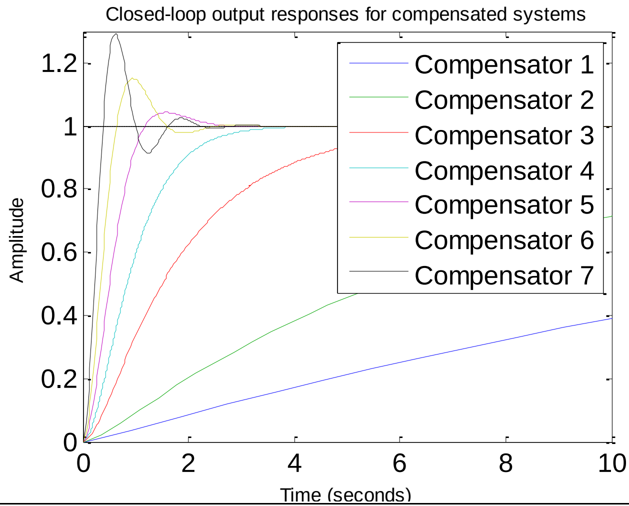


EXAMPLE 2:

$$\left\{G(s)=\frac{1}{s^2+4s};\quad M(s)=K\right\}$$
$$\Rightarrow\quad p_c=s^2+4s+K$$

$$K=0.2,0.5,2,4,8,15,30$$

As gain increases, the response is faster. If the gain is too high, overshoot ensures.

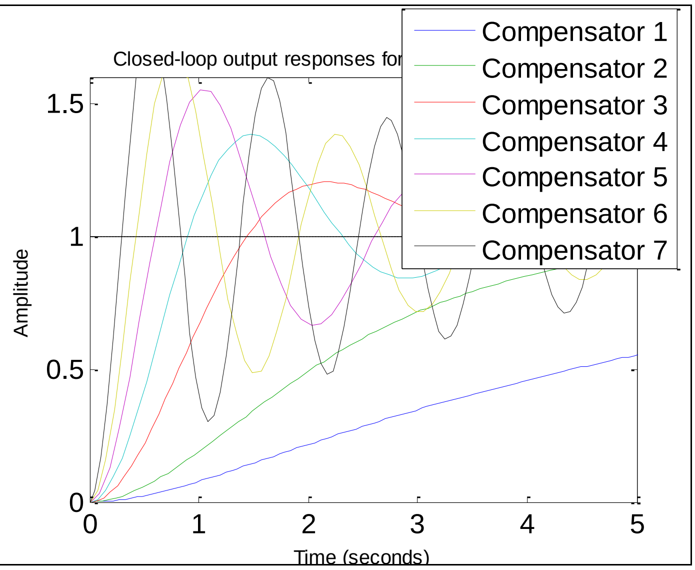


EXAMPLE 3:

$$\left\{G(s)=\frac{s+5}{s^3+5s^2+6s};\quad M(s)=K\right\}$$
$$\Rightarrow\quad p_c=s^3+5s^2+6s+[s+5]K$$

$$K=0.2,0.5,2,4,8,15,30$$

As gain increases, the response is increasingly oscillatory. The best K is neither too small nor too large (around K=1).



Exercises: Compare the closed-loop behaviour for various values of K

$$\left\{\begin{array}{l}G=\dfrac{1}{s^2+3s+2}\\ M(s)=\dfrac{0.4}{s}K\end{array}\right\};\quad \left\{\begin{array}{l}G=\dfrac{1}{s^3+3s^2+3s+1}\\ M(s)=\dfrac{0.2(s+3)}{(s+4)}K\end{array}\right\};\quad \left\{\begin{array}{l}G=\dfrac{1}{s^3+6s^2+11s+6}\\ M(s)=\dfrac{0.2}{s}K\end{array}\right\}$$