Modelling and control summaries

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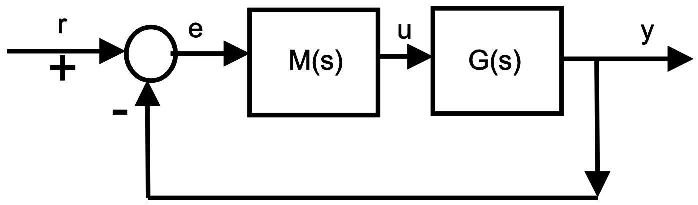
Root-loci 5: Introduction to sketching

This brief summary assumes readers are familiar with the concept of feedback and block diagrams. For simplicity the focus is on the simplest form of block diagram, although it is relatively straightforward to extend the concepts to more complex arrangements. Hence consider a process *G(s)* and a compensator *M(s)* which is expressed as a gain K multiplied by a transfer function. The closed-loop transfer function is $G_c(s)$.



$$GK\tilde{M} = K\frac{n(s)}{d(s)} \quad \Rightarrow \quad G_c = \frac{Kn}{\underbrace{Kn+d}_{p_c}}$$

$$M(s) = K\tilde{M}(s)$$

$$G_c = \frac{GK\tilde{M}}{1+GK\tilde{M}}$$

It helps to separate the controller gain from the dynamic parts so the closed-loop pole polynomial can be written down explicitly in terms of this gain K. **For simplicity, hereafter, we may use the shortcut notation that:**

$$G\tilde{M} \equiv G = K\frac{n}{d}$$

Definitions of closed-loop poles

It is useful to be aware of the numerous alternative definitions as they can all be useful.

$$p_c = 0 \quad \equiv \quad \left\{ \begin{array}{c} Kn + d = 0 \\ K\frac{n}{d} = -1 \\ \arg\left(\frac{n}{d}\right) = \pm 180^o \end{array} \right\}$$

$$p_c = 0 \quad \Rightarrow \quad \left|K\frac{n}{d}\right| = 1$$

$$\arg(n) - \arg(d) = 180^o$$

Determine the closed-loop pole polynomial for the following systems.

$G = \frac{1}{s^2+3s+2}$ $M(s) = \frac{0.4}{s}K$	$G = \frac{1}{s^3+3s^2+3s+1}$ $M(s) = \frac{0.2(s+3)}{(s+4)}K$	$G = \frac{1}{s^3+6s^2+11s+6}$ $M(s) = \frac{0.2}{s}K$
$p_c = s(s^2+3s+2)+0.4K$	$p_c = (s^3+3s^2+3s+1)(s+4)+0.2K(s+3)$	$p_c = s(s^3+6s^2+11s+6)+0.2K$

Use the phase condition to determine whether there exists a value of K such that a given value of 's' can be a closed-loop pole

This condition is easy to work with if all the polynomials are factorised.

$\left\{ \begin{array}{l} G = \dfrac{1}{(s+1)(s+2)} \\ M(s) = \dfrac{0.4}{s}K \end{array} \right\} \Rightarrow \begin{array}{c} n = 0.4 \\ d = s(s+1)(s+2) \end{array}$	Hence, there exists K such that s is a closed-loop pole if: $\angle n(s) - \angle d(s) = -\angle(s+1) - \angle(s+2) - \angle s = -180^o$
Clearly one can see that values such as s=-0.5 satisfy this condition and values such as s=1 do not.	

$\left\{ \begin{array}{l} G = \dfrac{1}{(s+1)^3} \\ M(s) = \dfrac{0.2(s+3)}{s+4}K \end{array} \right\} \Rightarrow \begin{array}{c} n = 0.2(s+3) \\ d = (s+1)^3(s+4) \end{array}$	There exists K such that s is a closed-loop pole if: $\angle n(s) - \angle d(s) = -3\angle(s+1) - \angle(s+4) + \angle(s+3) = -180^o$
Clearly one can see that values such as s=-2 satisfy this condition and values such as s=2 do not.	

REMARK:

One can write down the phase of a single factor by inspection **when 's' is real**.

$$\{\angle(s+a) = 0 \quad s > -a\} \quad \{\angle(s+a) = \pm 180^o \quad s < -a\}$$

We return to this later.