Modelling and control summaries

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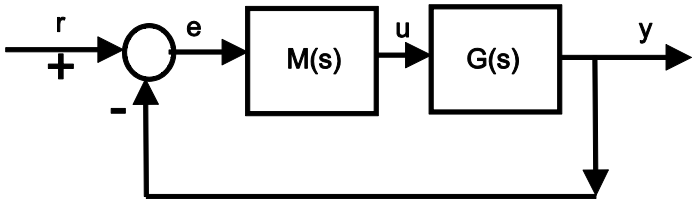
Root-loci 6: Start and end points

The focus is on the simplest form of block diagram, a process *G(s)* and a compensator *M(s)* which is expressed as a gain K multiplied by a transfer function. The closed-loop transfer function is $G_c(s)$.



$$M(s) = K\tilde{M}(s) \qquad G_c = \frac{GK\tilde{M}}{1+GK\tilde{M}} = \frac{Kn}{Kn+d}$$

Conditions for closed-loop poles

$$p_c = 0 \quad \equiv \quad \left\{ \begin{array}{c} Kn + d = 0 \\ K\dfrac{n}{d} = -1 \\ \arg\left(\dfrac{n}{d}\right) = \pm 180^o \end{array} \right\}$$

START POINTS of root-loci correspond to when K=0	
1. Poles are given by *Kn(s)+d(s)=0*. 2. If K=0, poles are given by *d(s)=0*. **3.** *d(s)=0* defines the open-loop poles.	• **Loci must begin from open-loop poles.** • **It is conventional to mark open-loop poles with an `X' symbol.**

END POINTS of root-loci correspond to when K ➔ ∞	
1. Poles are given by *Kn(s)+d(s)=0*. 2. Let K➔∞, and assume some of the poles are finite. If s is finite, then d(s) must be finite, hence Kn(s) is finite. 3. If K➔∞ and Kn(s) is finite then n(s)➔0. 4. n(s)=0 defines attraction points for closed-loop poles with large K.	• **Some loci must end at the open-loop zeros.** • **It is conventional to mark open-loop zeros with an `O' symbol.**

SUMMARY

Loci begin at open loop poles and some finish at open-loop zeros.

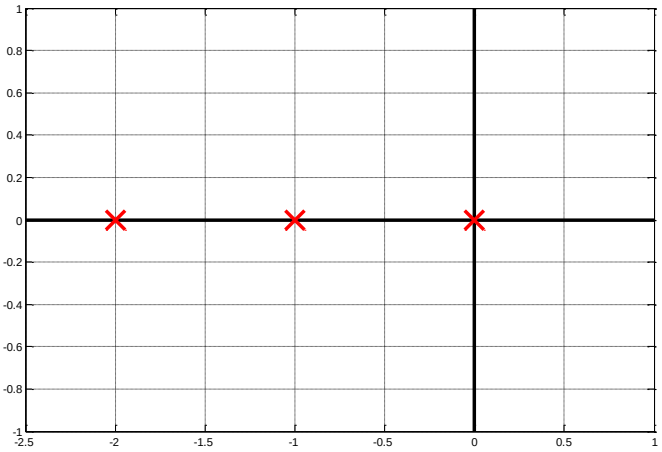
Illustrations: Mark the start and end points for the following systems

$$G = \frac{1}{s^2 + 3s + 2}$$

$$M(s) = \frac{0.4}{s} K$$

Poles at 0, -1,-2

No zeros



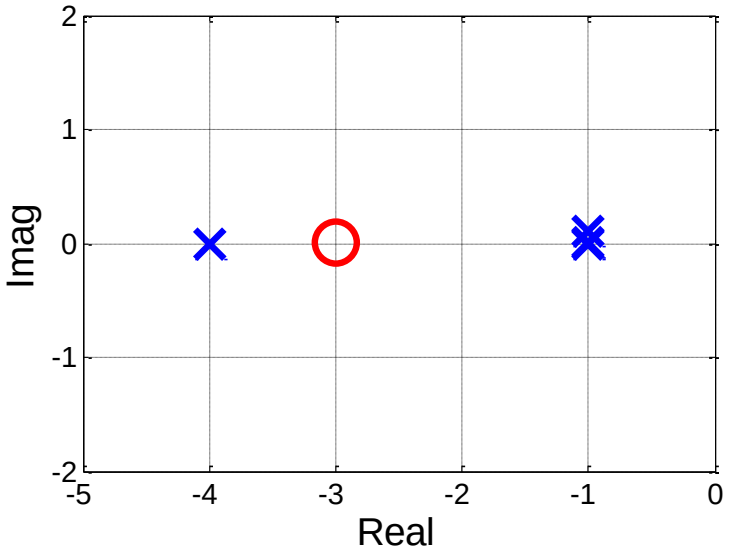
$$G = \frac{1}{s^3 + 3s^2 + 3s + 1}$$

$$M(s) = \frac{0.2(s+3)}{(s+4)} K$$

Poles at -1, -1,-1,-4

Zero at -3

NOTE: Where a pole/zero is multiple, must mark each occurrence clearly!



$$G = \frac{1}{s^3 + 6s^2 + 11s + 6}$$

$$M(s) = \frac{0.2}{s} K$$

Poles at 0, -1,-2,-3

No Zeros