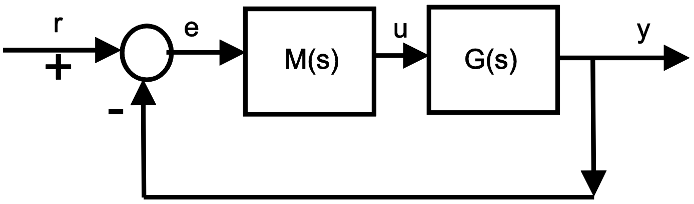
Modelling and control summaries

by Anthony Rossiter

Root-loci 7b: Asymptote centroid

The focus is on the simplest form of block diagram, a process *G(s)* and a compensator *M(s)* which is expressed as a gain K multiplied by a transfer function. The closed-loop transfer function is $G_c(s)$.



$$M(s)=K\tilde{M}(s)$$

$$G_c=\frac{GK\tilde{M}}{1+GK\tilde{M}}=\frac{Kn}{Kn+d}$$

Conditions for closed-loop poles

$$p_c=0 \quad \equiv \quad \left\{ \begin{array}{c} Kn+d=0 \\ K\dfrac{n}{d}=-1 \\ \arg\left(\dfrac{n}{d}\right)=\pm 180^o \end{array} \right\}$$

REMARK: The centroid is only defined if the excess of poles over zeros is at least 2.

Rules of polynomials	
For **monic** polynomials, the 2nd coefficient is minus the sum of the roots.	
$p(s)=(s+a)(s+b)(s+c)(s+d)$ $p(s)=s^4+(a+b+c+d)s^3+....+abcd$	**If the 2nd coefficient is fixed, then the sum of the roots must also be fixed!**

Closed-loop pole polynomial has a fixed 2nd coefficient if k-m>=2

$$\left. \begin{array}{l} n=n_m s^m+n_{m-1}s^{m-1}+\cdots n_0 \\ d=d_k s^k+d_{k-1}s^{k-1}+\cdots d_0 \end{array} \right\} \Rightarrow \quad p_c=Kn+d=d_k s^k+d_{k-1}s^{k-1}+\cdots Kn_0+d_0$$

1. **If k-m>=2, then the sum of the closed-loop poles is fixed ($-d_{k-1}$) and also**
2. **The sum of the closed-loop poles is the same as the sum of the open-loop poles.**

SUMMARY (for k-m>=2):

1. **k-m poles go to the asymptotes (sum must be (k-m)*centroid of asymptotes).**
2. **m poles go to the open-loop zeros**
3. **Sum(closed-loop poles) =Sum(open-loop zeros)+(k-m)*centroid of asymptotes**

Key Observation:
Sum(open-loop poles) = Sum(closed-loop poles) Sum(closed-loop poles)= Sum(open-loop zeros) +(k-m)*Centroid HENCE Centroid = [Sum(open-loop poles) –Sum(open-loop zeros)]/(k-m)

Find the asymptotes CENTROID for the following

$G = \frac{1}{s^2+3s+2}$ $M(s) = \frac{0.4}{s}K$	3 poles so k=3. No zeros so m=0. Hence k-m=3. Sum(open-loop poles)=-3. Sum(open-loop zeros)=0 **Centroid = [-3 – 0]/3 = -1**

$G = \frac{1}{s^2+3s+2}$ $M(s) = 0.5K$	2 poles so k=2. No zeros so m=0. Hence k-m=2. Sum(open-loop poles)=-3. Sum(open-loop zeros)=0 **Centroid = [-3 – 0]/2 = -1.5**

$\left\{ \begin{array}{c} G = \frac{1}{s^3+6s^2+11s+6} \\ K(s) = \frac{(s+6)}{(s+4)}K \end{array} \right\}$	4 poles so k=4. 1 zero so m=1. Hence k-m=3. Sum(open-loop poles)=-10. Sum(open-loop zeros)=-6 **Centroid = [-10 –(-6)]/3 = -4/3**

$\left\{ \begin{array}{c} G = \frac{1}{s^2+11s+30} \\ K(s) = \frac{(s+1)}{(s+2)}K \end{array} \right\}$	3 poles so k=3. 1 zeros so m=1. Hence k-m=2. Sum(open-loop poles)=-13. Sum(open-loop zeros)=-1 **Centroid = [-13 +1]/2 = -6**

$\left\{ \begin{array}{c} G = \frac{1}{s^3+6s^2+11s+6} \\ K(s) = \frac{(s+6)}{s(s+4)}K \end{array} \right\}$	5 poles so k=5. 1 zeros so m=1. Hence k-m=4. Sum(open-loop poles)=-3. Sum(open-loop zeros)=0 **Centroid = [-10+6]/4 = -1**

<u>REMARK</u>: If there is only 1 asymptote, then there is no centroid.