Modelling and control summaries

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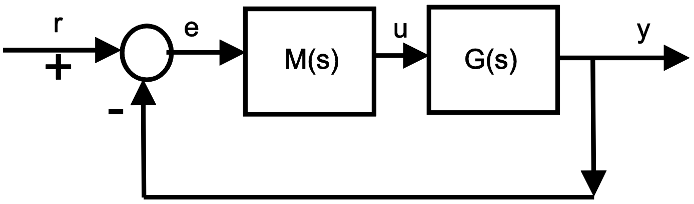
Root-loci 7: Asymptote directions

The focus is on the simplest form of block diagram, a process *G(s)* and a compensator *M(s)* which is expressed as a gain K multiplied by a transfer function. The closed-loop transfer function is $G_c(s)$.



$$M(s) = K\tilde{M}(s)$$

$$G_c = \frac{GK\tilde{M}}{1+GK\tilde{M}} = \frac{Kn}{Kn+d}$$

Conditions for closed-loop poles

$$p_c = 0 \quad \equiv \quad \left\{ \begin{array}{c} Kn + d = 0 \\ K\frac{n}{d} = -1 \\ \arg\left(\frac{n}{d}\right) = \pm 180^o \end{array} \right\}$$

Asymptotic END POINTS of root-loci corresponding to when K → ∞

Poles are given by *Kn(s)+d(s)=0*. As K→∞, some poles go to the open-loop zeros, the remainder go to asymptotes at infinity. We are looking for values of 's' such that:

$$\left. \begin{array}{l} s \to \infty \\ K \to \infty \end{array} \right\} \quad \Rightarrow \quad Kn(s) + d(s) = 0$$

Key Observations about large numbers

$$\left. \begin{array}{c} n = n_m s^m + n_{m-1}s^{m-1} + \cdots n_0 \\ s \to \infty \end{array} \right\} \Rightarrow \begin{array}{c} n(s) \approx n_m s^m \\ s^m >> s^{m-1} \end{array}$$

$$\left. \begin{array}{c} d = d_k s^k + d_{k-1}s^{k-1} + \cdots d_0 \\ s \to \infty \end{array} \right\} \Rightarrow \begin{array}{c} d(s) \approx d_k s^k \\ s^k >> s^{k-1} \end{array}$$

$$\left. \begin{array}{l} s \to \infty \\ K \to \infty \end{array} \right\} \Rightarrow \quad Kn + d \to Kn_m s^m + d_k s^k; \quad Kn_m s^m + d_k s^k = 0 \quad \Rightarrow \quad (Kn_m + d_k s^{k-m}) = 0$$

$$s = \sqrt[k-m]{-Kn_m / d_k}$$

SUMMARY: Asymptotic directions are (k-m)th roots of -1.

$s = \sqrt[k-m]{-Kn_m / d_k}$	$k-m=2 \Rightarrow \sqrt{-1} = \pm 90^o$ $k-m=3 \Rightarrow \sqrt[3]{-1} = -180^0, \pm 60^0$ $k-m=4 \Rightarrow \sqrt[4]{-1} = \pm 45^0, \pm 135^o$

Find the asymptotic directions of the root-loci for the following

$G = \frac{1}{s^2+3s+2}$ $M(s) = \frac{0.4}{s}K$	3 poles so k=3 No zeros so m=0 Hence k-m=3	$\sqrt[3]{-1} = -180^0, \pm 60^0$

$G = \frac{1}{s^2+3s+2}$ $M(s) = 0.5K$	2 poles so k=2 No zeros so m=0 Hence k-m=2	$\sqrt[2]{-1} = \pm 90^0$
$\left\{ \begin{array}{l} G = \frac{1}{s^3+6s^2+11s+6} \\ K(s) = \frac{(s+6)}{(s+4)}K \end{array} \right\}$	4 poles so k=4 1 zeros so m=1 Hence k-m=3	$\sqrt[3]{-1} = -180^0, \pm 60^0$
$\left\{ \begin{array}{l} G = \frac{1}{s^2+11s+30} \\ K(s) = \frac{(s+1)}{(s+2)}K \end{array} \right\}$	3 poles so k=3 1 zeros so m=1 Hence k-m=2	$\sqrt[2]{-1} = \pm 90^0$

$\left\{ \begin{array}{l} G = \frac{1}{s^3+6s^2+11s+6} \\ K(s) = \frac{(s+6)}{s(s+4)}K \end{array} \right\}$	5 poles so k=5 1 zeros so m=1 Hence k-m=4	$\sqrt[4]{-1} = \pm 45^0, \pm 135^0$

<u>REMARK</u>: If there is only 1 asymptote, then the direction is -180^o.

$\left\{ \begin{array}{l} G = \frac{s+4}{s^2+5s+6} \\ K(s) = \frac{(s+6)}{(s+4)}K \end{array} \right\}$	3 poles so k=3 2 zeros so m=2 Hence k-m=1	$\sqrt[1]{-1} = -1 \equiv -180^0$