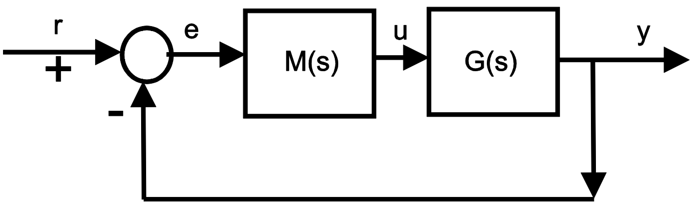
Modelling and control summaries

by Anthony Rossiter

Root-loci 8: Real axis on the loci

The focus is on the simplest form of block diagram, a process *G(s)* and a compensator *M(s)* which is expressed as a gain K multiplied by a transfer function. The closed-loop transfer function is $G_c(s)$.



$$M(s) = K\widetilde{M}(s)$$

$$G_c = \frac{GK\widetilde{M}}{1+GK\widetilde{M}} = \frac{Kn}{Kn+d}$$

Conditions for closed-loop poles

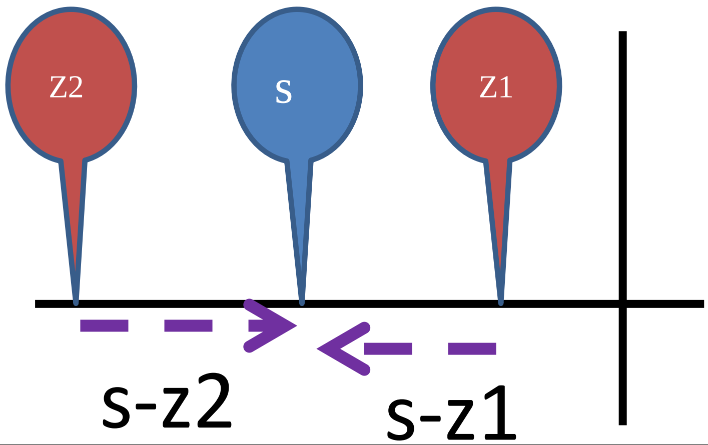
$$p_c = 0 \quad \equiv \quad \left\{ \begin{array}{c} Kn + d = 0 \\ K\frac{n}{d} = -1 \\ \arg\left(\frac{n}{d}\right) = \pm 180^o \end{array} \right\}$$

Determine the argument of (n/d) when s is on the real axis

For a single factor (s-z) make the following observation, assuming both s and z are real.

$$s > z \quad \Rightarrow \quad \angle(s-z) = 0$$
$$s < z \quad \Rightarrow \quad \angle(s-z) = 180$$

If s is <u>to the right</u> of a root z, the argument of factor (s-z) is ZERO.
If s is <u>to the left</u> of a root z, the argument of factor (s-z) is 180°.



Clearly as Z2 is to the left of s, the resulting argument of (s-Z2) is 0.

Z1 is to the right of s and so the resulting argument of (s-Z1) is 180°.

WE ignore complex factors/roots as their respective arguments cancel out for s on the real axis.

Phase of a complex number product is given by sum of phases of individual terms
$$\angle(abcd) = \angle a + \angle b + \angle c + \angle d$$
1. Phase of n(s) is sum of phases of individual factors 2. Phase of d(s) is sum of phases of individual factors. 3. If s is real, we can ignore factors with complex roots as the resultant phase is 0.

Determine the argument of (n/d) when s is on the real axis

$$n = (s - z_1)(s - z_2)\cdots(s - z_m) \quad \Rightarrow \quad \angle n = \sum \angle(s - z_i)$$

$$d = (s - p_1)(s - p_2)\cdots(s - p_k) \quad \Rightarrow \quad \angle d = \sum \angle(s - p_i)$$

Phase of n(s) is the number of zeros to the right of s multiplied by 180°.
Phase of d(s) is the number of poles to the right of s multiplied by 180°.

$\angle n - \angle d =$ **(Number of poles + zeros to the right)*180°.**

$\angle n(s) - \angle d(s) = (180 \pm n360)^o \quad \Rightarrow$	**There must exist a K such that Kn/d=-1 and hence that value of s is a possible closed-loop pole.**

SUMMARY: If there are a odd number of (poles+zeros) to the right, then that part of the real axis must lie on the loci.

Mark the parts of the real axis on the loci.

$$G = \frac{1}{s^2 + 3s + 2}$$

$$M(s) = \frac{0.4}{s} K$$