Modelling and control summaries

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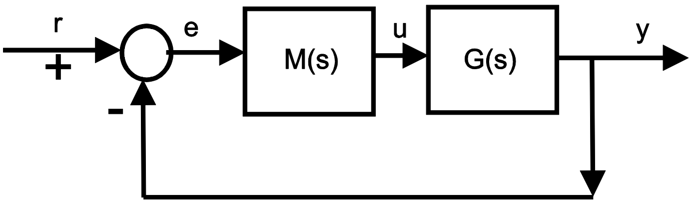
Root-loci 9: Sketching using 5 basic rules

The focus is on the simplest form of block diagram, a process *G(s)* and a compensator *M(s)* which is expressed as a gain K multiplied by a transfer function. The closed-loop transfer function is $G_c(s)$.



$$M(s) = K\tilde{M}(s)$$

$$G_c = \frac{GK\tilde{M}}{1+GK\tilde{M}} = \frac{Kn}{Kn+d}$$

Conditions for closed-loop poles

$$p_c = 0 \quad \equiv \quad \left\{ \begin{array}{c} Kn + d = 0 \\ K\frac{n}{d} = -1 \\ \arg\left(\frac{n}{d}\right) = \pm 180^o \end{array} \right\}$$

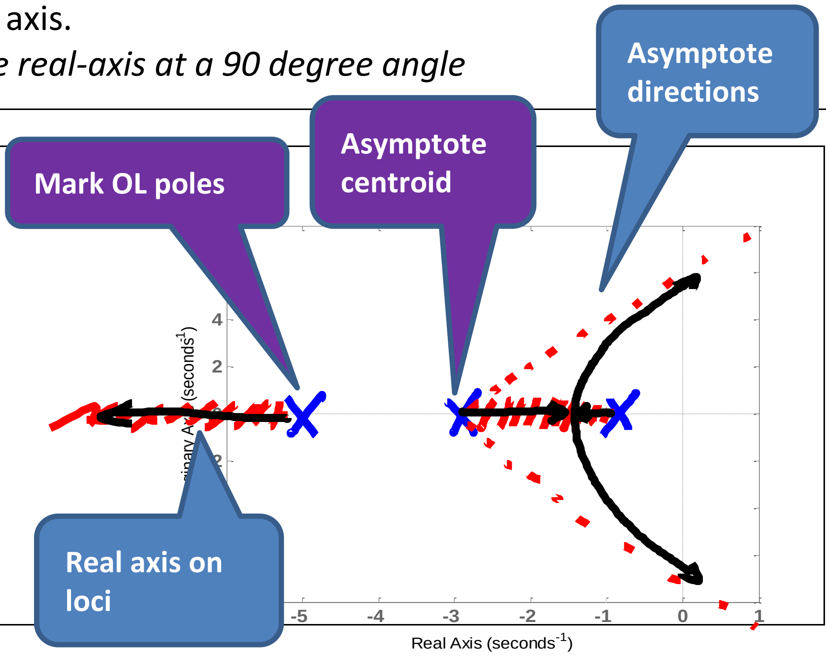
Summary of rules for sketching

Rule 1: Mark OL poles with a X
Rule 2: Mark OL zeros with a O
Rule 3: Compute asymptote directions from excess poles over zeros.
Rule 4: Compute asymptote centroid and add asymptotes to plot.
Rule 5: Add parts of loci on real axis.
Note that loci always depart the real-axis at a 90 degree angle

Apply rules 1 to 5.

$$G = \frac{1}{(s+1)(s+3)(s+5)}$$
$$M(s) = K$$

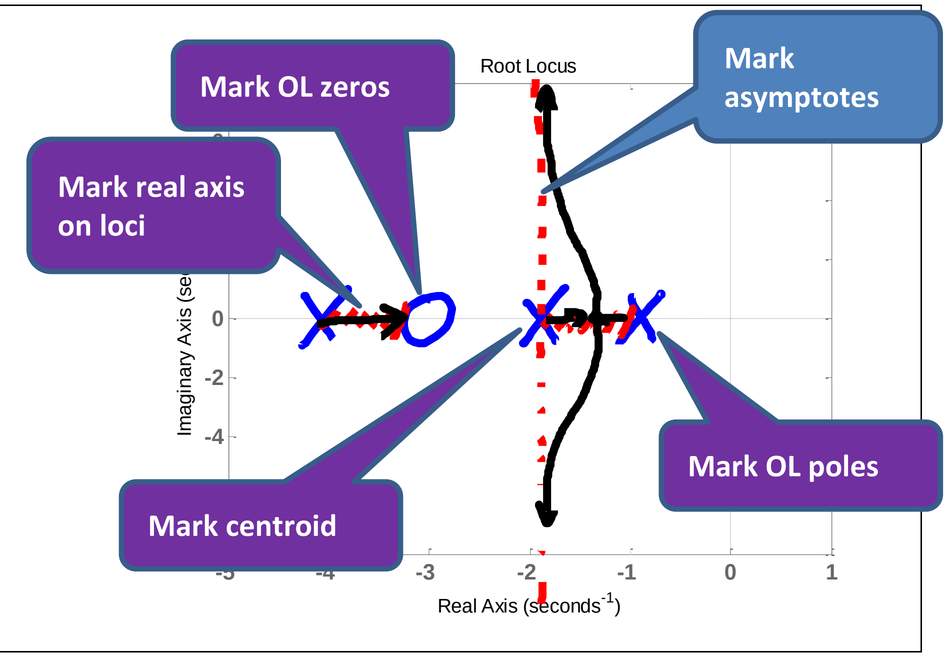
3 asymptotes
Centroid =-9/3=-3
Asymptote directions are minus 180 and plus/minus 60.



Apply rules 1 to 5.

$$G = \frac{s+3}{(s+1)(s+2)(s+4)}$$
$$M(s) = K$$

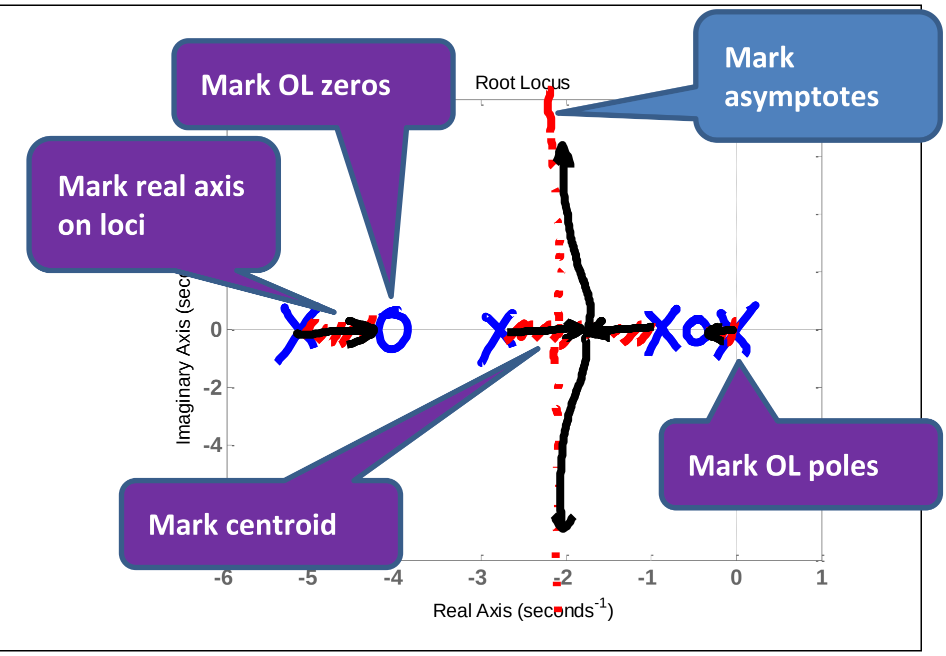
2 asymptotes
Centroid =-4/2=-2
Asymptote directions are plus/minus 90.



Apply rules 1 to 5.

$$G = \frac{s+0.5}{(s+1)(s+3)(s+5)}$$
$$M(s) = K\frac{(s+4)}{s}$$

2 asymptotes
Centroid =-4.5/2=-2.25
Asymptote directions are plus/minus 90.



Examples for students to try (use rlocus.m on MATLAB to test your answers)

$$\left\{ G = \frac{(s+2)}{(s+10)(s+3)(s+1)s} \quad M(s) = K \right\}$$

$$\left\{ G = \frac{s+3}{(s+1)^2} \quad M(s) = \frac{K}{s} \right\}$$

$$\left\{ G = \frac{20}{s(s+2)(s+3)} \quad M(s) = K\frac{s+1}{s+10} \right\}$$

$$\left\{ G = \frac{(s+3)}{(s+1)(s+2)(s+4)} \quad M(s) = K \right\}$$

$$\left\{ G = \frac{s+1}{s((s+2)^2+1)} \quad M(s) = K \right\}$$

$$\left\{ G = \frac{s-3}{s(s+2)(s+1)} \quad M(s) = K\frac{s+4}{s+0.5} \right\}$$