Example of GM

Find the GM for $G = \dfrac{1}{s(s+1)(s+2)}$

Procedure is:

- GM: Find freq where arg(G)=-180, then find gain.

Find phase cross-over frequency:

1

Example of GM 2

Find the GM for $G = \dfrac{2}{s(s+3)(s+4)}$

Procedure is:

- GM: Find freq where arg(G)=-180, then find gain.

2

Example of PM

Find the PM for $G = \dfrac{1}{s(s+0.2)}$

Procedure is:

- PM: Find freq. where |G|=1, then find phase.

Find freq. w_g where |G|=1.

Finally, PM=180+arg(G)

3

Example of PM 2

Find the PM for $G = \dfrac{10}{(s+1)(s+2)}$

Procedure is:

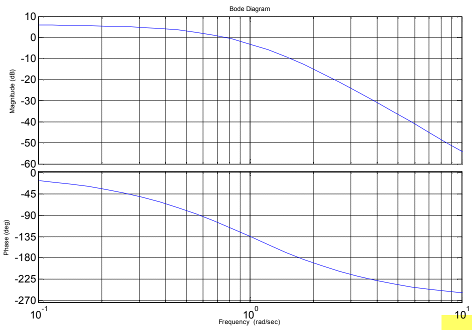
- PM: Find freq. where |G|=1, then find phase.

You may need to estimate.

4

Calculate the maximum positive gain K for closed loop stability with $G(s) = K \big/ s(s+1)^3$

Estimate margins from following Bode



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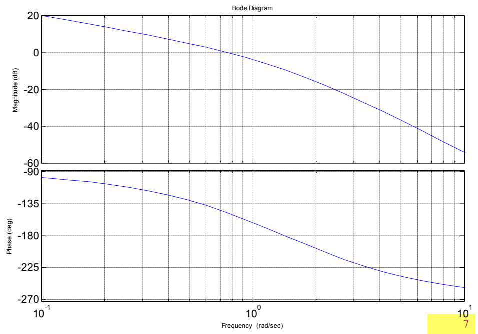
Estimate margins from following Bode



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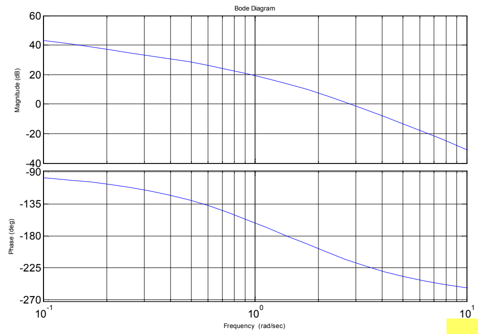
Estimate margins from following Bode



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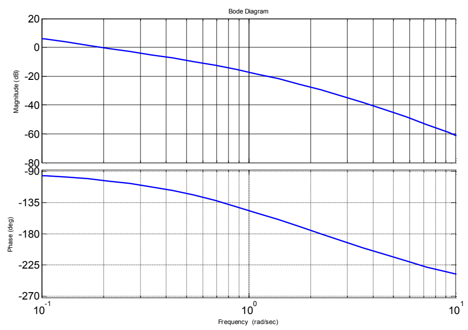
Example G=1/[s(s+1)(s+5)]

Obtain the GM and PM for K=2, 10 and K=100.



$$G(s)=\frac{10}{s(1+0.1s)^2} \qquad M(s)=\frac{70}{s(s+4)(s+9)}$$

1. The open-loop transfer function of a unity-feedback system is given by G(s). Find the gain margin, the phase margin from the Bode diagrams (both sketched using asymptotic methods).
   Verify, numerically, that the gain cross over frequency is 6.82 rad/s and the phase cross over frequency is 10 rad/s and hence compute the gain and phase margins analytically. Contrast to the estimates from your sketches. Is this difference important ?
2. A chemical reactor plant can be modelled as a unity-feedback system whose open-loop transfer function is given by M(s). Sketch the Bode diagram and hence estimate the gain margin, the phase margin, the gain crossover frequency and the phase crossover frequency respectively. Verify that the gain/phase cross over frequencies are 1.75rad/s and 6rad/s respectively and hence find the GM and PM analytically.

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Consider a unity negative feedback system with forward path transfer function G where K is a proportional control system gain.

$$G(s)=\frac{K(1-s)}{(s+1)^2}$$

Sketch the form of the Nyquist plot of G(s) and use MATLAB to calculate it when K=1.

Use the Nyquist stability criterion to verify that the closed loop system is stable for gains in the range $K_{max} = 2 > K > -1 = K_{min}$

Sketch the form of the Nyquist Diagrams in both closed-loop stable and unstable cases.

Find the gain K such that the closed loop system phase margin is exactly 45^o. Use the Nyquist stability criterion to confirm that the closed-loop system is stable and calculate the corresponding gain margin in dB.

$$G(s)=\frac{K}{s(1+0.5s)(1+0.1s)} \qquad M(s)=\frac{10}{s(s+2)}$$

1. The open-loop transfer function of a unity feedback system is given by G(s). You are given that the GM is 12 when K=1. Hence find the gain margin when (a) K=5; (b) K=20. In each case, estimate the PM and comment on the stability of the closed-loop system.
2. Consider a feedback control system with process M(s). (a) With Find the gain margin and phase margin of the uncompensated system. What is the gain crossover frequency? (b) Now include a phase-lead compensator K= 2(s+2)/(s+4). What is the new phase margin ? What is the new gain crossover frequency? (c) Contrast the open-loop bandwidth before and after compensation. (d) Next, use a phase-lag compensator K = 0.5(s+0.4)/(s+0.2). What is the new phase margin ? What is the new gain crossover frequency? Contrast the open-loop bandwidth before and after compensation with the lead and lag compensators. (e) What can you conclude?

1. Consider the feedback system with process G(s) and compensator K(s). (a) Sketch the bode and nyquist plots and hence find the gain and phase margins with unity compensation ? (b) Add a lead compensator K(s) = 3(s+0.2)/(s+1.2). Sketch the new Bode plot and hence computed the new GM/PM. (c) Contrast the compensated and uncompensated systems. (d) Could you stabilise this system with a lag compensator ? $G = \frac{1}{s^2(s+5)}$

2. Consider the system M = 50/[s(s+1)(s+5)]. What are the GM and PM ? What is the affect of a lag compensator K(s)=0.2(s+0.2)/(s+0.04) ?

These two questions are better suited to MATLAB and are designed to improve your insight.

A DC servo example

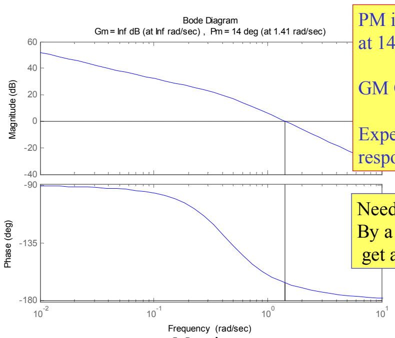
A DC servo G(s) is used to control position with a controller K(s).

Discuss the efficacy of the control system by investigating margins from the Bode plot:

$$G = \frac{1}{3s^2+2s+0.5}; \quad K = \frac{6s+2}{s}$$

Margins 14

Bode plot of servo system



PM is too small at 14 degrees

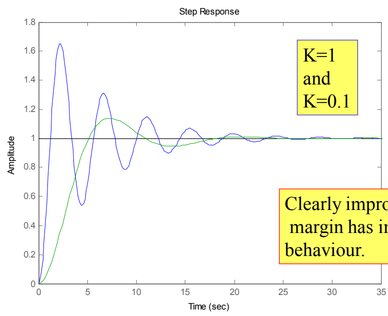
GM OK

Expect oscillatory response

Need to reduce gain By a factor of 10 to get a PM of 52 degrees

Margins 15

Closed-loop step responses



K=1 and K=0.1

Clearly improving phase margin has improved behaviour.

Margins 16

Examples

1. For $G=K/(s^2+s+1)$, find GM and PM with K=1 and find K so that PM=45
2. For $G=K/(s+1)^4$, find range of K so closed-loop stable and find K so that PM=60.
3. Calculate the maximum positive gain K for closed loop stability with $G(s) = \frac{K}{s(s+1)^3}$

Sketch solution

For $G=K/(s^2+s+1)$,

a) find GM and PM with K=1.
b) Find K so that PM=45

$$\angle G = -180 \Rightarrow \angle(1-w^2+jw) = -180 \Rightarrow w_p = \infty \Rightarrow GM = \infty$$

$$|G(jw)| = 1 \Rightarrow [1-w^2]^2 + w^2 = 1 \Rightarrow w_g = 1 \Rightarrow PM = 180 + \angle\frac{1}{jw} = 90$$

PROCEDURE:

First find gain and phase cross over frequencies

$$\angle G(j\omega_g') = 45 - 180 = -135$$

$$\frac{\omega_g'}{1-\omega_g'^2} = \tan(135^\circ) = -1 \Rightarrow \omega_g' = \frac{1+\sqrt{5}}{2} = 1.62 (rad/s)$$

Force gain cross over freq to be desired value

$$|G(j\omega_g')| = \frac{K}{\sqrt{(1-\omega_g'^2)^2 + \omega_g'^2}} = \frac{K}{2.3} \Rightarrow K = 2.3$$

Margins 18

Sketch solution

For G=K/(s+1)⁴

Find range of K so closed-loop stable.

Find K so that PM=60.

FIRST: Find phase cross over frequency and then ensure gain*GM less than one.

SECOND: find freq when phase is -120 and set gain to be one.

$$\angle G = -4\angle(jw+1); \quad \angle G = -180 \Rightarrow w = 1$$

$$GM = \frac{1}{|G(jw)|} = \sqrt{(w^2+1)^4} = 4$$

$$\angle G = -120 \Rightarrow \angle(jw+1) = 30 \Rightarrow w = \frac{1}{\sqrt{3}}$$

$$|KG(jw)| = 1 \Rightarrow \frac{K}{(w^2+1)^2} = \frac{9K}{16} = 1$$

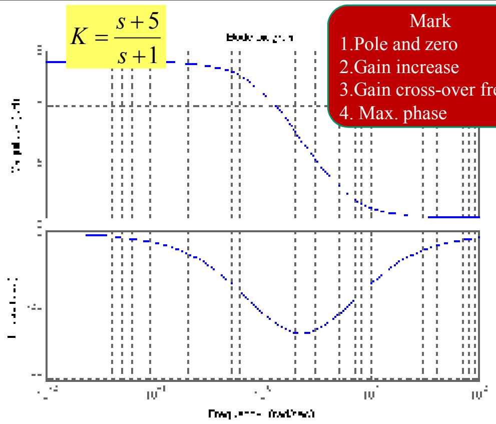
Margins 19

Delays (independent work)

- Delays give a limit to the gain that can be used.
- Explain this:
  - Using Nyquist
  - Using common sense.
- Delays in MATLAB
  - H=tf([1], [1 1],'ioDelay',1)

Margins 20

$$K = \frac{s+5}{s+1}$$



Mark
1.Pole and zero
2.Gain increase
3.Gain cross-over freq.
4. Max. phase

Sketch the bode plot for K=2(s+4)/(s+1)

Mechanistic rules for lag design

(use these for the computer quiz but not for Daniel)

- Find the frequency w where argument is -120.
- Choose the gain K so that K|G(jw)|=1, thus w becomes the gain cross over freq.
- Choose the required gain recovery at low frequency, say H (typically about 5 for lab), then

$$K_{lag}(s) = K\frac{s + w/10}{s + w/10H}$$

Note factor of 10 reduction from gain cross over frequency.

Write down on paper before putting in sisotool to avoid silly errors.

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Do a lag design

$$G = \frac{10(s+2)}{s(s+1)(s+4)^2}$$

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Student examples

Implement the following system and lag compensator pairs on MATLAB and contrast with simple gain compensation.

$$G(s)=\frac{40}{s(s+2)};\quad K(s)=\frac{1}{10}\left(\frac{\frac{1}{7}+s}{\frac{1}{70}+s}\right);\quad K=1;\quad K=0.1$$

$$G=\frac{5}{s(s+1)(0.5s+1)};\quad K=\frac{1}{30}\frac{s+0.02}{s+0.004};\quad K=1;\quad K=\frac{1}{30}$$

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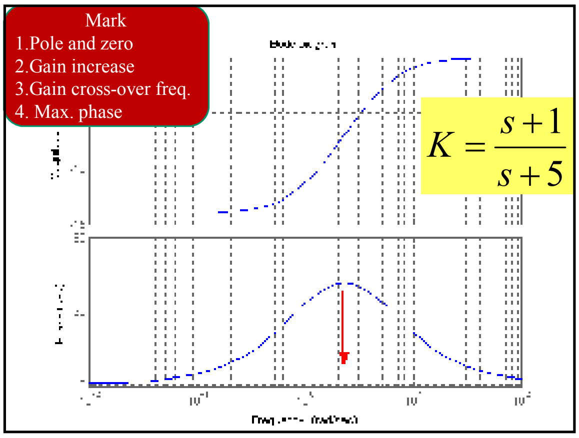

$$K=\frac{s+1}{s+5}$$

Sketch the bode plot for K=2(s+0.2)/(s+1)

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Mechanistic rules for lead design

- Choose the desired gain cross-over frequency w. (The best choice is no always obvious but in the lab it is taken as twice that used by the lag compensator.)
- Find rotation 'a' required so that arg(G(jw)) +a =-120.
- Use a lookup table to find the value β for a lead with maximum rotation 'a'.
- Choose the gain of the lead so that w is the gain cross over frequency of the compensated system, i.e.

β	2	3	4	6	8	10
Max. phase	19	30	37	46	51	55

$$K_{lead}(s)=\frac{\sqrt{\beta}}{|G(jw)|}\frac{s+w/\sqrt{\beta}}{s+w\sqrt{\beta}}$$

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Student examples

Implement lead and lag compensator pairs on MATLAB and contrast with simple gain compensation.

$$G(s)=\frac{40}{s(s+2)};\qquad G=\frac{5}{s(s+1)(0.5s+1)}$$

We are particularly interested in the differences in bandwidth (roughly the gain cross over frequency), speed of response, input activity (aggressive or not).

Which would you choose and why?

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Overlaying responses and the assessed laboratory

Generally we want to compare the behaviour of several strategies, for instance K1, K2, K3.

We should know how to overlay the important plots (Bode, Nyquist, closed-loop input/output) in order to allow easy comparison.

You need to export compensators from sisotool to the workspace.

bode(G*K1,G*K2,G*K3);legend('K1','K2','K3')
nyquist(G*K1,G*K2,G*K3);legend('K1','K2','K3')
overlaymany(G,K1,K2,K3)

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Lead properties

A simple optimisation gives this result.

At what frequency is the maximum phase?

$$\angle K_{lead} = -\tan^{-1}\frac{w}{\beta a} + \tan^{-1}\frac{w}{a}; \quad \max \angle K_{lead} \Rightarrow \quad \omega_m = \sqrt{\beta a^2}$$

What is this maximum phase. Substitute in w_m, so

$$\min \angle K_{lead} = -\tan^{-1}\frac{\sqrt{\beta a^2}}{\beta a} + \tan^{-1}\frac{\sqrt{\beta a^2}}{a} = -\tan^{-1}\frac{1}{\sqrt{\beta}} + \tan^{-1}\sqrt{\beta}$$

$$= -\tan^{-1}\frac{\frac{1}{\sqrt{\beta}} - \sqrt{\beta}}{2}$$

Depends only on ratio of pole to zero, not their actual positions..

β	2	3	4	6	8	10
Max. phase	19	30	37	46	51	55

Observation

- For unstable systems such as some aerospace systems, high gain control is necessary.
- In these cases lag will be inappropriate and lead is essential.
- Consider the system $G = \dfrac{1}{s(s-1)}$
- PM is negative for all values of gain so simple changes in gain cannot achieve a positive PM. Only positive phase rotation can achieve this!
- The same insight is obvious from root-loci. Try it!

Try this on MATLAB and get help in a lab session if necessary.