

The open-loop transfer function of a unity-feedback system is given by $G(s)$. Find the gain margin, the phase margin from the Bode diagrams (both sketched using asymptotic methods).

$$G(s) = \frac{10}{s(1+0.1s)^2}$$

Verify, numerically, that the gain cross over frequency is 6.82 rad/s and the phase cross over frequency is 10 rad/s and hence compute the gain and phase margins analytically. Contrast to the estimates from your sketches. Is this difference important ?

A chemical reactor plant can be modelled as a unity-feedback system whose open-loop transfer function is given by $M(s)$. Sketch the Bode diagram and hence estimate the gain margin, the phase margin, the gain crossover frequency and the phase crossover frequency respectively. Verify that the gain/phase cross over frequencies are 1.75rad/s and 6rad/s respectively and hence find the GM and PM analytically.

$$M(s) = \frac{70}{s(s+4)(s+9)}$$

The open-loop transfer function of a unity feedback system is given by $G(s)$. You are given that the GM is 12 when $K=1$. Hence find the gain margin when (a) $K=5$; (b) $K=20$. In each case, estimate the PM and comment on the stability of the closed-loop system.

$$G(s) = \frac{K}{s(1+0.5s)(1+0.1s)}$$

Consider a feedback control system with process $M(s)$. (a) Find the gain margin and phase margin of the uncompensated system. What is the gain crossover frequency? (b) Now include a phase-lead compensator $K = \frac{2(s+2)}{(s+4)}$. What is the new phase margin ? What is the new gain crossover frequency? (c) Contrast the open-loop bandwidth before and after compensation. (d) Next, use a phase-lag compensator $K = \frac{0.5(s+0.4)}{(s+0.2)}$. What is the new phase margin ? What is the new gain crossover frequency? Contrast the open-loop bandwidth before and after compensation with the lead and lag compensators. (e) What can you conclude?

$$M(s) = \frac{10}{s(s+2)}$$

Consider the feedback system with process $G(s)$ and compensator $K(s)$. (a) Sketch the bode and nyquist plots and hence find the gain and phase margins with unity compensation ? (b) Add a lead compensator $K(s) = \frac{3(s+0.2)}{(s+1.2)}$. Sketch the new Bode plot and hence computed the new GM/PM. (c) Contrast the compensated and uncompensated systems. (d) Could you stabilise this system with a lag compensator ?

$$G = \frac{1}{s^2(s+5)}$$

Consider the system $M = 50/[s(s+1)(s+5)]$. What are the GM and PM ? What is the affect of a lag compensator $K(s)=0.2(s+0.2)/(s+0.04)$?

A DC servo $G(s)$ is used to control position with a controller $K(s)$.

$$G = \frac{1}{3s^2 + 2s + 0.5}; \quad K = \frac{6s + 2}{s}$$

Discuss the efficacy of the control system by investigating margins from the Bode plot:

1. What are the key characteristics of lead and lag and how are these are used. Find some example designs from the text books to illustrate your points.
 2. Discuss under what circumstances you would use a lag and when you would use a lead. What are the advantages and disadvantages of each compensator ? Why would you use a lead-lag compensator?
 3. Practise implementing some designs on MATLAB and checking whether the effect of compensation agrees with your expectations.
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For unstable systems such as some aerospace systems, high gain control is necessary. In these cases lag will be inappropriate and lead is essential. Demonstrate this with some examples of your own including one for which PM is negative for all compensators that comprise a simple scalar; in these cases simple changes in gain cannot achieve a positive PM so what would you do? Don't forget to include insights from root-loci.